Interpolation from Curves in Pseudoconvex Boundaries

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Introduction

Let D be a smoothly bounded pseudoconvex domain in $\mathbf{C}^n$, and let $A^\infty(D)$ denote those smooth functions on $\bar{D}$ that are holomorphic on D. (Throughout, smooth is synonymous with infinitely differentiable.) Recall that a compact subset K of ∂D is said to be an *interpolation set* for $A^\infty(D)$ if each smooth function on K can be extended to an element of $A^\infty(D)$. In this paper we are interested in conditions on a given smooth submanifold M of ∂D ensuring that each compact subset of M is an interpolation set. For strongly pseudoconvex domains this problem is well understood (see, e.g., [6] and [4]); in this case the natural condition that M be complex-tangential is known to be sufficient for interpolation. (Recall that M is said to be *complex-tangential* if for each $p \in M$ the tangent space $T(M,p)$ is contained in the maximal complex subspace of $T(\partial D, p)$.) In addition to this, in the general case a necessary condition for interpolation is that ∂D satisfy a finite-type condition along M (namely, that complex hypersurfaces have bounded order of contact with ∂D at points of M); for this, see the argument in [10, Ex. 4.1]. We work in $\mathbf{C}^2$ and, in view of the above conditions, assume that M is 1-dimensional. Here are our main results.

THEOREM. *Let D be a smoothly bounded pseudoconvex domain of finite type in $\mathbf{C}^2$, and let $M \subset \partial D$ be a smooth complex-tangential curve.*

(1.2) *If ∂D is of constant type along M, then every compact subset of M is an interpolation set for $A^\infty(D)$.*

(2.1) *If ∂D and M are real-analytic, then for each $p \in M$ there exists a neighborhood V of p so that every compact subset of $M \cap V$ is an interpolation set for $A^\infty(D \cap V)$.*

We remark that the formulation of finite type used here can be found in [5, Lecture 28, p. 121].

Our proof of the first result above depends on the following theorem. Recall that a subset K of ∂D is called a *peak set* for $A^\infty(D)$ if there exists

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