Complemented Ideals in Weighted Algebras of Analytic Functions on the Disc

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Introduction

Weighted algebras of analytic functions introduced in a more general setting by Hörmander [2] have been investigated for many years with different aims. During the last years in the case of entire functions, several authors have considered the question of whether closed ideals are complemented subspaces of the algebra regarded as a locally convex space (see e.g. Taylor [14]; Meise [7]; Meise and Taylor [9], [10]; Meise and Vogt [11]; Meise, Momm, and Taylor [8]). We deal with the case of analytic functions on the disc and give a criterion to decide whether an arbitrary given ideal is complemented.

Let $(\psi_k)_{k \in \mathbf{N}}$ be an increasing sequence of continuous, nonnegative, strictly increasing, unbounded functions on $x \geq 1$. By $A_\mathbf{P}$, we denote the locally convex space of all analytic functions f on the open unit disc satisfying some estimate

$$|f(z)| \leq C \exp \psi_k\left(\frac{1}{1-|z|}\right), \quad |z|<1.$$

We require conditions on the weight functions to guarantee that $A_\mathbf{P}$ becomes an algebra of analytic functions in which division is possible. We characterize the complemented closed ideals in $A_\mathbf{P}$ by their distribution of zeros: *A nonzero closed ideal I is complemented in $A_\mathbf{P}$ if and only if there exists $m \in \mathbf{N}$ such that for each $k \in \mathbf{N}$ there is $n \in \mathbf{N}$ with*

$$\psi_k\left(\frac{1}{1-|a|}\right)\psi_n^{-1}\left(\psi_k\left(\frac{1}{1-|a|}\right)\right) \leq \frac{1}{1-|a|}\psi_m\left(\frac{1}{1-|a|}\right)$$

for almost all zeros a of I.

We also obtain a corresponding result for another (dual) type of weighted algebras which can be defined similarly to $A_\mathbf{P}$.

Via duality theory, our results can be regarded as results on the existence of continuous linear right inverses for Toeplitz operators, or (more generally) on the complementation of shift invariant subspaces in certain locally convex spaces of analytic functions on the disc which are smooth up to the boundary (see Momm [12]).

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