Free Duals and Regular Sequences

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Introduction

The first part of this paper is devoted to a study of the following question. Let A be a local Noetherian ring, let I be an ideal of A of finite projective dimension ($\mathrm{pd}_A(I) < \infty$), and let H_1 be the first Koszul homology module associated to a system of generators of I. If the dual module $(H_1)^* = \mathrm{Hom}_{A/I}(H_1, A/I)$ is A/I-free, then is I generated by a regular sequence? We obtain an affirmative answer in some cases, for example, when I/I^2 is torsion-free as an A/I-module.

In the second part we consider an analogous problem for the conormal module. Let K be a field, let R be a smooth K-algebra of essentially finite type, and let $B = R/J$. If $(J/J^2)^* = \mathrm{Hom}_B(J/J^2, B)$ is a projective B-module, what consequences are brought on B? A particular case of a conjecture of Vasconcelos [12] asserts that B must be a locally complete intersection. Under some additional hypotheses, we construct an exact sequence

$$0 \to H_1(K,B,B) \to J/J^2 \to (J/J^2)^{**} \to \Omega_{B|K} \to (\Omega_{B|K})^{**} \to \mathrm{Ext}_B^2(H^1(K,B,B),B) \to 0,$$

and we use it to obtain some evidence for the conjecture.

Finally we give a condition, in terms of Hochschild cohomology, for a locally complete intersection algebra to be regular.

We will use some properties of André–Quillen (co)homology $H(A,B,-)$ (see [1], [5], [8]).

1. On the Freeness of the Dual of the First Koszul Homology

Let A be a local Noetherian ring, let I be an ideal of A, and let H_1 be the first Koszul homology module associated to a system of n generators of I. Let $\alpha: H_1 \to (H_1)^{**}$ and $\beta: I/I^2 \to (I/I^2)^{**}$ be the canonical homomorphisms into the bidual module. We start by obtaining an exact sequence which relates the homomorphisms α and β.

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