On the Integrability of Double Walsh Series with Special Coefficients

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1. Introduction

We study the double Walsh series

$$\sum_{j=0}^{\infty} \sum_{k=0}^{\infty} a_{jk} w_j(x) w_k(y), \tag{1.1}$$

where $\{a_{jk}: j, k=0,1,\dots\}$ is a null sequence of real numbers; that is,

$$a_{jk} \to 0 \quad \text{as } \max(j,k) \to \infty \tag{1.2}$$

and $\{w_j(x): j=0,1,\dots\}$ is the well-known Walsh orthonormal system defined on the interval $I=[0,1)$ and taken in the Paley enumeration (see, e.g., [2]). Thus, series (1.1) is considered on the unit square $I^2=[0,1)\times[0,1)$.

The pointwise convergence of series (1.1) is usually meant in Pringsheim's sense. (See, e.g., [5, vol. 2, ch. 17].) In other words, we form the rectangular partial sums

$$s_{mn}(x,y) = \sum_{j=0}^{m-1} \sum_{k=0}^{n-1} a_{jk} w_j(x) w_k(y) \quad (m, n \ge 1),$$

then let both m and n tend to ∞, independently of one another, and assign the limit $f(x,y)$ (if it exists) to the series (1.1) as its sum.

Throughout this paper, we shall use the notations

$$\Delta_{10} a_{jk} = a_{jk} - a_{j+1,k}, \qquad \Delta_{01} a_{jk} = a_{jk} - a_{j,k+1},$$

$$\Delta_{11} a_{jk} = a_{jk} - a_{j+1,k} - a_{j,k+1} + a_{j+1,k+1} \quad (j, k \ge 0).$$

We say that $\{a_{jk}\}$ is a monotone decreasing sequence if a_{jk} is monotone decreasing in both j (for each fixed $k \ge 0$) and k (for each fixed $j \ge 0$), or equivalently

$$\Delta_{10} a_{jk} \ge 0 \quad \text{and} \quad \Delta_{01} a_{jk} \ge 0 \quad (j, k \ge 0) \tag{1.3}$$

We say that $\{a_{jk}\}$ is a sequence of bounded variation if

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