

Entire Timelike Minimal Surfaces in $E^{3,1}$

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1. Introduction

Calabi was the first to show that an entire *spacelike* minimal surface in Minkowski 3-space $E^{3,1}$ must be a plane (see [1]). However, if w is the timelike coordinate in u, v, w -space, the example

$$v = w \tanh u$$

shows that an entire *timelike* minimal surface in $E^{3,1}$ need not even be flat. The best one can say in this direction is that the surface must be conformally equivalent to the Minkowski 2-plane $E^{2,1}$ (see [6]).

In this paper we generate examples that display considerable variety in the shapes of entire timelike minimal surfaces in $E^{3,1}$. This is done, in part, by describing an analog for the classical construction of associate minimal surfaces in Euclidean 3-space $E^{3,0}$.

Associate minimal surfaces in $E^{3,0}$ are paired in an amusing manner. At corresponding points, they share the same induced metric, Gauss curvature, zero mean curvature, and unit normals. Still, associate minimal surfaces in $E^{3,0}$ can have markedly different shapes, as the helicoid and the catenoid amply illustrate. (For pictures, see [2] or [8].)

To produce associate families of spacelike minimal surfaces from a given spacelike minimal surface in $E^{3,1}$, the original classical construction suffices. But an entirely different construction must be used to generate associate families of timelike minimal surfaces from a given timelike minimal surface in $E^{3,1}$. In both cases, the associate pairing still preserves the Minkowski induced metric, Gauss curvature, zero mean curvature, and unit normals.

While it is pleasing to have the counterpart for a construction based on complex analytic techniques in a situation governed by the wave equation rather than by Laplace's equation, the construction of associate timelike minimal surfaces in $E^{3,1}$ is further justified by the fact that all surfaces associate to an entire timelike minimal surface in $E^{3,1}$ are entire over the same fixed plane. Thus the construction can be used to produce infinite families of isometric entire, timelike minimal surfaces in $E^{3,1}$ no two of which are congruent.

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