Parareflexive Operators on Banach Spaces

MATJAŽ OMLADIČ

1. Introduction

Nilpotent operators on finite-dimensional spaces are all direct sums of Jordan cells. In infinite-dimensional Hilbert spaces this is not true in general; however, every nilpotent operator there is still quasi-similar to a direct sum of Jordan cells [1]. A slightly different approach yields quasi-similarity of an arbitrary nilpotent operator on a Hilbert space to a direct sum of Jordan models, that is, Jordan block-cells [1]. The latter approach seems more appropriate for transplanting these results into a Banach space. This is accomplished in Section 2, where we apply the notion of quasi-similarity that was extended to Banach spaces in [9].

The results of Section 2 are applied in Section 3 to extend to Banach spaces a Hilbert space result on parareflexive operators [1]. Unfortunately, some of our results are proven only in Banach spaces that satisfy a technical condition (see condition (A) introduced in §2). This condition is satisfied in particular by all Hilbert spaces and by all separable Banach spaces. Our main result is a characterization of parareflexive operators on Banach spaces that satisfy condition (A).

2. Nilpotent Operators

The most simple nilpotent operators on a Banach space are *Jordan operators,* that is, operators $J_m(X)$ acting on a direct sum $X^m = X \oplus X \oplus \cdots \oplus X$ of m copies of a Banach space X, where m is a positive integer supplied with (say) the l_1 norm and defined by

$$J_m(X)(x_1, x_2, \dots, x_m) = (x_2, x_3, \dots, x_m, 0).$$

Observe that this operator can be represented by an operator matrix

$$J_m(X) = \begin{bmatrix} 0 & I & 0 & \dots & 0 & 0 \\ 0 & 0 & I & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & 0 & I \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

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