Finite Group Actions on the Moduli Space of Self-Dual Connections, II

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1. Introduction

Let G be a finite group, and let M be a simply connected, closed, smooth 4-dimensional manifold with a positive definite intersection form and a smooth action of G on it. Let $\Pi: E \to M$ be a quaternion line bundle with instanton number one and with a G-action on E through bundle isomorphism such that Π is a G-map. Let $\mathfrak{M}$ be the set of self-dual connections on E modulo the group $\mathcal{G}$ of gauge transformations. If we use a G-invariant metric on M then the moduli space $\mathfrak{M}$ is a G-space, but $\mathfrak{M}$ might not be a manifold because of the nonvanishing second cohomology group of the fundamental elliptic complex or because of reducible self-dual connections.

In [5] Donaldson used a compact perturbation of a Fredholm map to make $\mathfrak{M}$ a manifold. In [7] Freed and Uhlenbeck proved that for generic metric on M the moduli space $\mathfrak{M}$ is a manifold. We cannot use their methods directly to make the G-set $\mathfrak{M}$ into a G-manifold, because the perturbation cannot be made G-invariant and so the method of [7] need not yield a G-invariant metric.

In [4] we defined cohomology classes which are obstructions to perturbing the G-set $\mathfrak{M}$ into a G-manifold. In this paper we shall show that when G is the cyclic group of order 2^n, there are classes of metrics on M for which these obstruction classes vanish.

We will follow the notations in [4]; $\hat{}$ stands for irreducibility. Let $\mathcal{C}$ be the set of all connections on E and let $\mathcal{G}$ be the group of gauge transformations on E. Consider the map $\Phi: \mathcal{C}^\wedge \times C^G \to \Omega^2_-(\mathcal{G}_E)$ given by $\Phi(\nabla, \psi) = P_- \psi^{-1*} R^\nabla$, where $C^G = C^k(GL(TM))^G$ is the set of G-equivariant C^k-automorphisms of the tangent bundle of M. Here $P_-: \Omega^2(\mathcal{G}_E) \to \Omega^2_-(\mathcal{G}_E)$ is the projection to the anti-self-dual part (with respect to a fixed G-invariant metric on M) of the 2-forms of M with values in the adjoint bundle associated to E, and R^∇ denotes the curvature of the connection ∇. Our result is that there is an open G-set O of $\mathcal{C}^\wedge \times C^G$ such that the restriction map $\Phi: O \to \Omega^2_-(\mathcal{G}_E)$ is smooth and has zero as a regular value. The G-set O contains all $(\nabla, \psi) \in \mathcal{C}^\wedge \times C^G$ such that $\Pi(\nabla) \in \mathfrak{M}^G$ with respect to the metric $\psi^*(g)$ on M, where $\Pi: \mathcal{C}^\wedge \to \mathcal{B}^\wedge = \mathcal{C}^\wedge/\mathcal{G}$ is the projection. Furthermore, there is an

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