Restriction to Transverse Curves of Some Spaces of Functions in the Unit Ball

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Introduction

Let B denote the unit ball in C^n, and S its boundary. Let $h^\infty(B)$ denote the space of bounded pluriharmonic functions in B, and let $H^\infty(B)$ be the subspace of $h^\infty(B)$ of holomorphic functions. Also, for $\alpha<1$, we consider the algebra $\mathrm{Lip}_\alpha(B)$ of holomorphic functions in B, satisfying a Lipschitz condition of order α with respect to the Euclidean metric.

In this paper we deal with restrictions of these spaces to closed curves $\Gamma\subset S$. Here we will summarize the main results of the paper and introduce at the same time some of the required notations.

We will work with a simple (without intersections) periodic transverse curve, $\gamma: R\to S$ of class C^1. Recall that a curve is transverse if, for every t in R, $\gamma'(t)$ does not lie in the complex-tangent space $P_{\gamma(t)}$ at the same point. Analytically this condition is equivalent to the relation $\mathrm{Im}\,\gamma'(t)\overline{\gamma(t)}\neq 0$ (whereas $\mathrm{Re}\,\gamma'(t)\overline{\gamma(t)}=0$, simply because γ is on S). By choosing the reparametrization $s(t)=\int_a^t|\mathrm{Im}\,\gamma'(x)\overline{\gamma(x)}|\,dx$, $a\le t\le b$, where a and b satisfy $\gamma(a)=\gamma(b)$, we obtain a parametrization such that $\gamma'(t)\overline{\gamma(t)}=i$. With an appropriate dilation, we will suppose from now on that the curve is 2π-periodic, and there exists $\lambda>0$ such that, for all t, $\gamma'(t)\overline{\gamma(t)}=\lambda i$. In the following we will write I for $[-\pi,\pi]$, and $\Gamma=\gamma([-\pi,\pi])$.

We also consider the Koranyi pseudodistance $d(z,w)=|1-\bar{z}w|$, where $zw=\sum_i z_i w_i$. This defines a pseudodistance only on S, but we will consider it defined as well when one of the two variables is not in S.

In one complex variable, Fatou's theorem gives sense to the space $h^\infty{}_{|T}$ of boundary values of bounded harmonic functions, and the use of the Poisson transform shows that this space equals $L^\infty(T)$.

In several complex variables, a result of Nagel, Rudin, and Wainger (see [5] and [6]) states a Fatou type theorem implying the existence at almost every point of a C^1 transverse curve of the radial limit of a bounded holomorphic function; in fact, it proves the existence of a stronger kind of limit, the restricted K-limit. It is easy to see that the Nagel–Rudin–Wainger theorem holds for bounded pluriharmonic functions so that the space $h^\infty{}_{|\Gamma}$ is

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