Farrell and Mergelyan Sets for H^p Spaces $(0<p<1)$

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Dedicated to professor Ernst Selmer on the occasion of his 70th birthday

1. Introduction

Let F be a relatively closed subset of the open unit disc $\mathbf{D}$ of the complex plane $\mathbf{C}$, and let A be a space of analytic functions on $\mathbf{D}$ endowed with a topology τ such that the polynomials are τ-dense in A. The set F is said to be a *Farrell set for* (A,τ) if for each function $f\in A$ whose restriction $f|_F$ is bounded there exists a sequence $(p_\nu)_{\nu=1}^{\infty}$ of polynomials satisfying:

(1) $p_\nu\to f$ in the τ-topology, as $\nu\to\infty$,

(2) $p_\nu\to f$ pointwise on F, as $\nu\to\infty$, and

(3) $\|p_\nu\|_F\to\|f\|_F$, as $\nu\to\infty$.

As usual, $\|g\|_B$ denotes $\sup\{|g(z)|: z\in B\}$. Similarly, F is said to be a *Mergelyan set for* (A,τ) if, for each function $f\in A$ whose restriction to F is uniformly continuous, there exists a sequence $(p_\nu)_{\nu=1}^{\infty}$ of polynomials such that

(α) $p_\nu\to f$ in the τ-topology, as $\nu\to\infty$, and

(β) $p_\nu\to f$ uniformly on F, as $\nu\to\infty$.

Farrell and Mergelyan sets have been described for several cases: (a) A is the space $H^\infty(\mathbf{D})$ of all bounded analytic functions and τ the topology of pointwise convergence on $\mathbf{D}$ [9]; (b) A is the Hardy space H^p $(1\le p<\infty)$ and τ is the weak topology [8] or the norm topology [7]; (c) A is the space $H(\mathbf{D})$ of all analytic functions on $\mathbf{D}$ with the topology of uniform convergence on compact subsets of $\mathbf{D}$.

A holomorphic function in $\mathbf{D}$ is said to belong to the Nevanlinna class N if its characteristic function

$$T(r,f)=\frac{1}{2\pi}\int_0^{2\pi}\log(1+|f(re^{i\theta})|)\,d\theta$$

is bounded for $0\le r<1$. In this case $N(f)=\operatorname{Sup}_{0\le r<1}T(r,f)$. A function $f\in N$ is said to belong to the Smirnov class N^+ if there hold

$$\lim_{r\to 1}\int_0^{2\pi}\log(1+|f(re^{i\theta})|)\,d\theta=\int_0^{2\pi}\log(1+|f(e^{i\theta})|)\,d\theta.$$

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