Generators of Certain Groups of Semi-free S^1 Actions on Spheres and Splitting of Codimension-3 Knot Exact Sequences

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0. Introduction

Let $\Sigma_k^n(S^1)$ denote the set of oriented equivariant diffeomorphism classes of smooth semi-free S^1 actions on oriented homotopy $(n+2k-1)$-spheres satisfying these properties:

(P) The fixed point set is diffeomorphic to the standard $(n-1)$-sphere S^{n-1}. The normal bundle of the fixed point set is trivial as a complex vector bundle where the complex structure is an induced one from the action (see the Conventions below).

In fact, $\Sigma_k^n(S^1)$ is an abelian group under the equivariant connected sum operation (except for some low-dimensional cases). The group structure is fairly well understood. First, Hsiang [8] noted that $\Sigma_1^n(S^1)=0$. On the other hand it has been observed by many people that $\Sigma_k^n(S^1)$ is nontrivial in many cases; for instance, Browder [3] applied surgery theory to exhibit elements of infinite order for certain values of n and k. Later, Browder and Petrie [4] determined the rank of the free part of $\Sigma_k^n(S^1)$ as follows:

$$\text{(0.1)} \qquad \operatorname{rank}_Z \Sigma_k^n(S^1) = \operatorname{rank}_Z H^{4*}(CP^{k-1}\times(D^n,S^{n-1});Z)-\epsilon,$$

where $\epsilon=1$ if $n+2k-2\equiv 0\,(4)$ and $\epsilon=0$ otherwise. In particular it follows that $\operatorname{rank}_Z \Sigma_2^n(S^1)=1$ if and only if $n\equiv 0\,(4)$. In fact, $\Sigma_2^4(S^1)$ is known to be infinitely cyclic (i.e., torsion free).

Under these circumstances Davis [5, Prop. 7.15] has discovered that the generator of $\Sigma_2^4(S^1)$ is given by a semi-free smooth S^1 action defined naturally on an exotic 7-sphere discovered by Milnor [16]. An alternative proof is given in [13]. The result of Davis motivates this question:

What is a generator of the free part of $\Sigma_k^n(S^1)$? In other words, *is there an explicit description for such a generator?*

As is well known, famous Brieskorn spheres support natural semi-free smooth S^1 actions and some of them satisfy Property (P). We can verify that one of them is a generator or twice a generator of the free part of $\Sigma_k^n(S^1)$

Received November 10, 1987. Revision received September 23, 1988.
Michigan Math. J. 36 (1989).