The Existence of 7-fields and 8-fields on $(8k+5)$-dimensional Manifolds

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1. Introduction

Let M be a closed, connected and smooth manifold whose dimension n is congruent to 5 mod 8 with $n \geq 21$. Let η be a spin n-plane bundle over M. We shall investigate the span of η. Recall that the Kervaire mod 2 semi-characteristic of M, $\chi_2(M)$, is defined by

$$\chi_2(M) = \sum_{2i<n} \dim_{\mathbf{Z}_2} H^i(M;\mathbf{Z}_2) \bmod 2.$$

When η is the tangent bundle of M and M is 3-connected mod 2, we have from [12] that $\operatorname{span}(\eta) \geq 6$ if and only if $w_{n-5}(M) = 0$ and $\chi_2(M) = 0$, where $w_i(M)$ is the ith mod 2 Stiefel–Whitney class of M.

We shall prove the following theorems.

THEOREM 1.1. *If M is 5-connected* mod 2, *then* $\operatorname{span}(M) \geq 7$ *if and only if $\delta w_{n-7}(M) = 0$ and $\chi_2(M) = 0$, where δ is the Bockstein operator associated with the exact sequence $0 \to \mathbf{Z} \to \mathbf{Z} \to \mathbf{Z}_2 \to 0$.*

THEOREM 1.2. *Suppose M is 5-connected* mod 2 *and $Sq^1 H^{n-7}(M;\mathbf{Z}_2) = 0$. Then* $\operatorname{span}(M) \geq 8$ *if and only if $w_{n-7}(M) = 0$, $0 \in \psi_3(w_{n-9}(M))$, and $\chi_2(M) = 0$, where ψ_3 is a stable secondary cohomology operation associated with the relation*

$$\psi_3 : Sq^2 Sq^2 + Sq^1(Sq^2 Sq^1) = 0.$$

Some applications to immersions of manifolds into Euclidean spaces are given in the last section. Throughout the paper we assume that $\dim M = n$ is congruent to 5 mod 8 with $n \geq 21$. All cohomology will be ordinary cohomology with mod 2 coefficients unless otherwise specified.

2. The Modified Postnikov Tower

We shall consider the problem of finding an s-field as a lifting problem. Let $B\hat{SO}_j\langle 8\rangle$ be the classifying space of orientable j-plane bundles ξ satisfying $w_2(\xi) = w_4(\xi) = 0$, where $w_i(\xi)$ is the ith mod 2 Stiefel–Whitney class of

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