The Irreducibility of the 3-Sphere

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1. Introduction

In the theory of 3-dimensional manifolds constant use is necessarily made of the fact that S^3, the 3-dimensional sphere, is irreducible. This fact is usually required in its piecewise linear interpretation, for that seems to be the commonly chosen framework for elementary work with 3-manifolds. The required result is then the following "Schönflies theorem."

THEOREM. *If S^2 is embedded piecewise linearly in S^3, then $S^3 - S^2$ has two components, the closure of each being a piecewise linear ball.*

This theorem was proved by Alexander [1], and a version of his proof is given in [8]. That proof is not, however, readily understood in the context of the standard modern theory of piecewise linear n-manifolds, and the theorem is omitted from the main expositions of that theory ([3], [6], [9], [10]). It is likewise omitted from works on 3-manifolds (e.g., [5], [7]). The purpose of this paper is to give a version of the proof based on handlebody theory. It is hoped that this proof will fill a gap in the literature and that it will bring out the 3-dimensional nature of the proof (an innermost circle argument). That itself is of interest in that the Schönflies problem for S^3 embedded in S^4 is still unsolved in the piecewise linear or smooth sense; a discussion appears in Chapter 3 of [9]. (For locally flat embeddings of S^{n-1} in S^n the result is known to be true in the topological sense for all n [2], and, using the solution to the n-dimensional Poincaré conjecture, in the piecewise linear sense for $n \geq 5$.)

2. Piecewise Linear Preliminaries

A few easily accessible results of piecewise linear topology that will be needed are listed below.

(1) *An S^1, piecewise linearly embedded in S^2, separates S^2 into two piecewise linear discs.*

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