Composition Property of Holomorphic Functions on the Ball

BOO RIM CHOE

THEOREM. *Suppose* $\varphi \in A(B)$, *with* $\varphi(B) \subset U$, *is holomorphic across its maximum modulus set. Then* $g \circ \varphi \in \bigcap_{0<p<\infty} H^p(B)$ *for every Bloch function g on U. If, in addition,* $\{\varphi^m\}_{m=0}^{\infty}$ *forms an orthogonal set in* $H^2(B)$, *then there exists a weight* $\alpha = \alpha(\varphi)$ *such that* $h \circ \varphi \in H^p(B)$ *for every* $h \in A_\alpha^p(U)$ *and for every p* $(0<p<\infty)$.

This result will be easily derived from a careful analysis of the behavior of such a function φ near its maximum modulus set. For a class of functions φ we obtain the best possible weights $\alpha(\varphi)$. These are nonhomogeneous (even rational) functions, unlike the previous examples of P. Ahern, P. Russo, and the author.

1. Introduction

We will write $B = B_n$ for the open unit ball of $\mathbf{C}^n$ $(n \geq 1)$ and let $S = S_n = \partial B_n$. For $n=1$, we let $U = B_1$ and $T = S_1$; for further notation see Section 2. Throughout the paper $n \geq 2$ unless otherwise specified.

It has been known that the homogeneous polynomials

$$\varphi(z) = n^{n/2} z_1 \cdots z_n, \quad ([1])$$

$$\varphi(z) = z_1^2 + \cdots + z_n^2, \quad ([15])$$

$$\varphi(z) = b_\alpha z_1^{\alpha_1} \cdots z_n^{\alpha_n}, \quad ([2])$$

normalized so that $\varphi(B) = U$, have the following composition property:

If $g \in \mathfrak{B}(U)$, *the Bloch space on U, then* $g \circ \varphi \in \mathrm{BMOA}(B)$.

Here $\mathrm{BMOA}(B)$ denotes the space of holomorphic functions in $H^2(B)$ whose boundary functions are of bounded mean oscillations with respect to the non-isotropic balls on S (see [8]). These results have been recently generalized by the author. In [5] it is shown that the same property holds for every φ belonging to a certain class of holomorphic homogeneous polynomials. It is, however, still open whether the same holds for every holomorphic homogeneous polynomial that maps B onto U. It is known that

Received October 14, 1988.
Michigan Math. J. 36 (1989).