Regularity of Certain Rigid Isometric Immersions of n-dimensional Riemannian Manifolds into $\mathbf{R}^{n+1}$

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1. Introduction and Statement of Results

Let M be a real analytic (C^ω) Riemannian manifold and let F be an isometry of differentiability class C^1 of M onto another C^ω Riemannian manifold $\tilde{M}$. Then F is C^ω and uniquely determined by $F(P)$ and $dF(P)$ at a point $P \in M$. The reason is that F is locally a linear mapping between the normal coordinates of M near P and the normal coordinates of $\tilde{M}$ near $F(P)$. This uniqueness and analyticity of the isometries do not hold for the isometric immersions, as the following example shows.

EXAMPLE 1.1. Let $\gamma(s) = (\gamma^1(s), \gamma^2(s))$ be a plane curve parameterized by arclength s. If γ is C^∞ but not C^ω then the mapping $(s,t) \mapsto (\gamma^1(s), \gamma^2(s), t)$ is a C^∞ isometric immersion of $\mathbf{R}^2$ into $\mathbf{R}^3$, which is not C^ω. Furthermore, there is not uniqueness either; namely, an isometric immersion F of $\mathbf{R}^2$ into $\mathbf{R}^3$ cannot be determined by $F(P)$ and $dF(P)$ at a point $P \in \mathbf{R}^2$.

The author's question is whether an isometric immersion F is analytic if F is locally rigid. An isometric immersion $F: M \to \mathbf{R}^N$ is said to be locally rigid at $P \in M$ if, for any open neighborhood U of P, there exists an open set V such that $P \in V \subset U$ having the following property: If F' is any isometric immersion of V into $\mathbf{R}^N$ then there exists an isometry of $\mathbf{R}^N$ such that $F' = \tau \circ F$. Then the question is the following: Let M be a C^ω Riemannian manifold and let $F: M \to \mathbf{R}^N$ be an isometric immersion of class C^k, $k \gg 0$. Let $P \in M$. Then will F be C^ω at P if F is rigid at P? This paper is a partial answer to this question. Our main result is the following.

THEOREM 1.1. *Suppose that M is a C^ω Riemannian manifold of dimension $n \geq 3$ and $F: M \to \mathbf{R}^{n+1}$ is an isometric immersion of class C^2. Suppose that the immersed submanifold $F(M)$ has at least three nonzero principal curvatures at $F(P)$. Then F is C^ω at P.*

Note that the existence of three nonzero principal curvatures (the definition is recalled in Section 2) is a sufficient condition for $F(M)$ to be locally rigid,

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