Positive Entropy Homeomorphisms on the Pseudoarc

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Answering another of Marcy Barge's questions, we show that there is a pseudoarc homeomorphism with positive topological entropy. Since iterating a homeomorphism of positive entropy yields one of arbitrarily large entropy, it follows that the pseudoarc admits homeomorphisms of arbitrarily large entropy. Whether or not given a positive number r there is a pseudoarc homeomorphism of that entropy r is not known, and is not answered here, but is another of Marcy Barge's questions. As is often the case, in order to obtain this result we developed a tool which itself yields more information about the pseudoarc.

A *continuum* is a compact connected metric space. A continuum X is *homogeneous* if for $x, y \in X$ there is a space homeomorphism h such that $h(x) = y$. A continuum is *chainable* or *arclike* or *snakelike* if for each $\epsilon > 0$ there is a chain $C = \{C_0, \dots, C_n\}$ of open sets of diameter less than ϵ that covers X. C is a *chain* if $C_i \cap C_j \neq \emptyset$ if and only if $|i-j| \leq 1$. A pseudoarc, which is a nonseparating plane continuum, can be characterized as a homogeneous chainable continuum. Pseudoarcs, although arclike, contain no continuous nontrivial images of arcs, and in fact every nondegenerate subcontinuum of a pseudoarc is itself a pseudoarc. Another extraordinary fact about this continuum is that most continua [in the sense that they form a dense G_δ-set in the space of all continua (Vietoris topology)] in the plane are pseudoarcs.

A compact metric space is a *compactum*. A compactum X is *indecomposable* if every proper subcontinuum of X is nowhere dense in X. It is *hereditarily indecomposable* if every subcontinuum of X is itself indecomposable. The pseudoarc is a hereditarily indecomposable continuum.

For us, P will denote a pseudoarc, $I = [0, 1]$, $\mathbf{Z}$ is the integers, and $\mathbf{N}$ is the positive integers. If X is a compact metric space, $H(X)$ denotes its group of self homeomorphisms.

A chain $C = \{C_0, \dots, C_n\}$ is *taut* whenever $C_i \cap C_j \neq \emptyset$ if and only if $\bar{C}_i \cap \bar{C}_j \neq \emptyset$. A chain covers a set A *essentially* if there is a continuum Q contained in A such that each link contains a point of Q not in the closure of any other link. An open set o in a space X is *regular* if $\operatorname{Int} \bar{o} = o$. In the

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