LATTICES OF A LIE GROUP AND SEIFERT FIBRATIONS

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1. Introduction. Let L be a Lie group with finitely many components, K a maximal compact subgroup of L, and Λ a lattice of L. Λ acts properly discontinuously on the contractible manifold $K\backslash L$. The isotropy subgroups are finite and the orbit space $K\backslash L/\Lambda$ is an orbifold. If Λ is torsion-free, then the action of Λ is free and the orbit space is a manifold. The purpose of this article is to prove a structure theorem for $K\backslash L/\Lambda$; it roughly says that either it is a Riemannian orbifold of nonpositive sectional curvature or it Seifert fibers over such an orbifold. We do this if L satisfies the following extra condition $(*)$:

$(*)$ the center of $MR\backslash L_0$ is finite, where L_0 is the identity component of L, R is the radical of L, and M is the Lie subgroup of L_0 which corresponds to the sum of the compact simple factors of the semi-simple semi-direct summand of a Levi decomposition of the Lie algebra of L_0.

Without condition $(*)$, our construction still produces a Seifert fibration

$$K\backslash L/\Lambda \to O^m$$

over an orbifold O^m of dimension $m>0$. The condition $(*)$ is used to show that O^m has non-positive sectional curvature. If L is amenable, then $(*)$ is satisfied and it is not a restriction at all. The precise statement of the main theorem is:

THEOREM 1. *Let L be a non-compact Lie group with finitely many components satisfying $(*)$, K a maximal compact subgroup of L, and Λ a lattice of L. Then there is an orbifold Seifert fibration*

$$K\backslash L/\Lambda \to O^m,$$

where O^m is a Riemannian orbifold of dimension $m>0$ and of nonpositive sectional curvature. If L is amenable, O^m can be chosen to be flat.

REMARKS. (1) Condition $(*)$ is unnecessary if L is connected and Λ is uniform. See §4. (2) O^m is in general not a manifold, even when $K\backslash L/\Lambda$ is a manifold. (3) If Λ is only a discrete subgroup of L, our construction may not produce a Seifert fibration. We heavily use the lattice property of Λ. (4) Some special cases of Theorem 1 have been known; see Farrell and Hsiang [5; 6] and Quinn [12].

To begin the construction, we first choose a connected closed normal subgroup S of L. Then KS is closed, and we have a fiber bundle

$$K\backslash KS \to K/L \to KS\backslash L.$$

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