ISOMETRICALLY REMOVABLE SETS FOR FUNCTIONS IN THE HARDY SPACE ARE POLAR

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For a domain D in $\mathbf{C}^n$ and $0 < p < \infty$, $H^p(D)$ denotes the Hardy space of analytic functions $f: D \to \mathbf{C}$ for which $|f|^p$ has a harmonic majorant. If E is a relatively closed subset of D, then E is said to be a set of *removable singularities* (or E is said to be *removable*) for $H^p(D \setminus E)$ provided that $D \setminus E$ is connected and each f in $H^p(D \setminus E)$ has an analytic extension to a function in $H^p(D)$. This can be phrased in functional analysis terms by saying that E is a set of removable singularities for $H^p(D \setminus E)$ precisely when the restriction map $H^p(D) \to H^p(D \setminus E)$ is surjective. With this observation, say that E is *isometrically removable* if the restriction map $H^p(D) \to H^p(D \setminus E)$ is a surjective isometry. The main result of this paper is the characterization of isometrically removable sets as the polar sets (provided $D \setminus E$ supports a nonconstant function in H^p). In particular, this shows that isometric removability is independent of p.

The study of removable singularities for functions in a Hardy space does not originate with this paper. One of the first papers on Hardy spaces for arbitrary domains in $\mathbf{C}$ is [11], where (among other things) it is shown that if E has logarithmic capacity 0 and $E \subseteq D$, then E is removable for $H^p(D \setminus E)$. In [9] (compare [8]), as an extension of results of [1], it was shown that a relatively closed polar subset E of a domain D in $\mathbf{C}^n$ is a removable set of singularities for $H^p(D \setminus E)$. Järvi's proof [9] that polar sets are removable sets of singularities for the Hardy spaces actually shows that they are isometrically removable. The main contribution of this note, therefore, is that the converse holds. Indeed, the key to this converse is the first lemma below, which is a purely potential theoretic one. An application of this lemma will also be given to the study of removable singularities for the Hardy spaces h^p of harmonic functions, where the results do not exactly parallel those for the spaces H^p.

Before stating and proving the main results of this paper, it is advisable to collect some of the more crucial definitions as well as some relevant background information. For any p, if $f \in H^p(D)$, let u_f denote the least harmonic majorant of $|f|^p$. Fix a point a in D. For $1 \le p < \infty$, $\|f\|_p \equiv u_f(a)^{1/p}$ defines a norm on $H^p(D)$; for $p < 1$, $d(f,g) \equiv \|f-g\|_p^p = u_{f-g}(a)$ defines a metric on $H^p(D)$. The connectedness of D is necessary in order for $\|f\|_p$ to define a norm. (Though

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