THE LOCAL HULL OF HOLOMORPHY OF SEMIRIGID SUBMANIFOLDS OF CODIMENSION TWO

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1. Introduction. There has been considerable literature on determining the local hull of holomorphy for CR submanifolds of C^n. The case for real hypersurfaces was considered by Lewy in [11]. For generic submanifolds of higher codimension, it was shown in [10] that if the excess dimension of the Levi algebra at a point is maximal then the local hull of holomorphy contains an open set in C^n. Since that time, the goal of CR extension research has been (and still is) to more precisely determine the size of the local hull of holomorphy. This was done in [7] and later in [1] where it was shown that the cross section of the local hull of holomorphy "almost fills in" the convex hull of the image of the first Leviform at a point in the submanifold. The case when the first Leviform vanishes at a point, the relationship between the local hull of holomorphy and the second Leviform, has been given in [5], [6], and [9]. Recently in [2] it was shown that the local hull of holomorphy for a semirigid submanifold of higher type (to be defined below) contains a "wedge." This is a considerable improvement over the result in [10] alluded to above, which only guarantees that the local hull contains a cusplike set. However, the relationship between the size of the wedge and the type of the point was still missing. The goal of this paper is to explain this relationship for semirigid submanifolds of real codimension two.

Let us introduce notation. Suppose M is a smooth (C^k for k sufficiently large) CR submanifold of C^n of real codimension d. We let $T^C(M)$ and $H^C(M)$ be the complexified tangent bundle and complexified holomorphic tangent bundle respectively. We introduce the following tower of spaces. For $p \in M$, we let $L_p^0(M) = H_p^C(M)$ and in general we let $L_p^j(M)$ be the vector space spanned by $H_p^C(M)$ and all Lie brackets at p of elements in $H^C(M)$ up through length j. Note that the length of a Lie bracket is the number of vector fields in the bracket, that is, $\Lambda_m = [L_1, [L_2, \dots, [L_{m-1}, L_m], \dots]$ has length m. This notion of length is the same as that in [4] and length = order + 1 where order is the concept used in [8]. Note that

$$H_p^C(M) \subset L_p^j(M) \subset L_p^{j+1}(M) \subset T_p^C(M)$$

for all nonnegative j and in particular $2n-2d \le \dim_C L_p^j(M) \le 2n-d$.

DEFINITION 1.1 [4]. Let M be a generic submanifold of real codimension d ($d < n$) of C^n, and let $p \in M$. We say that p is a point of *type* $l = (l_1, \dots, l_d)$ where $l_1 \le l_2 \le \cdots \le l_d$ are integers if the following holds:

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