SEMILINEAR BOUNDARY VALUE PROBLEMS FOR UNBOUNDED DOMAINS

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1. Introduction. Let A be a non-negative self adjoint elliptic partial differential operator of order m on a (bounded or unbounded) domain $\Omega \subset \mathbf{R}^n$. We consider the Dirichlet problem for equations of the form

$$Au = f(x, u), \tag{1.1}$$

where $f(x, u)$ is a function defined on $\Omega \times \mathbf{R}$. Examples of the functions we consider include

$$f(x, u) = V(x)e^u(W(x)\cos e^u - 1), \tag{1.2}$$

where $V(x) \geq 0$, $V \in L^1$, $W \in L^\infty$. We show that for this choice of $f(x, u)$ the Dirichlet problem for (1.1) always has a solution (no matter what A, m, Ω are). The same is true for

$$f(x, u) = W(x) - V(x)ue^{u^2}, \tag{1.3}$$

where $W \in L^t$ for some t satisfying $1/2 \leq 1/t \leq 1/2 + m/2$ and V satisfies the assumptions above. Another example is

$$f(x, u) = V(x)[W(x)u^k \sin u^{k+1} - \sinh u + 1], \tag{1.4}$$

with V, W satisfying the same hypotheses as for (1.2) and $V \in L^t$ with t as above. We can also consider expressions such as

$$f(x, u) = W(x) - V(x)u^{2k-1}, \tag{1.5}$$

where V, W satisfy the same assumption as for (1.3).

In some instances we find a constant $\lambda_0 > 0$ such that

$$Au = \lambda f(x, u) \tag{1.6}$$

has a solution for each λ such that $0 < \lambda < \lambda_0$. This is done for the case

$$f(x, u) = V(x)|u|^q u + W(x), \tag{1.7}$$

where $q \geq -1$, $V \in L^\infty$, and $W \in L^t$ with $1/(q+2) + m/2 = 1/2 \leq 1/t \leq 1/2 + m/n$.

Another example is

$$f(x, u) = V_1(x)|u|^{q_1}u + V_2(x)|u|^{q_2}u, \tag{1.8}$$

with $-2 < q_1 < 0 < q_2$. In this case we give sufficient conditions for (1.6) to have a non-trivial solution.

We present two methods of attack. The first is to find a stationary point of a functional corresponding to (1.1). One of the major stumbling blocks in this

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