LAMINATIONS, FINITELY GENERATED PERFECT GROUPS, AND ACYCLIC MAPS

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1. Introduction. Let (M, N_1, N_2) denote a $(n+1)$-dimensional cobordism; that is, M is a compact, connected $(n+1)$-manifold with two boundary components N_1 and N_2. We investigate the conditions under which (M, N_1, N_2) admits a *lamination,* by which we mean an upper semicontinuous decomposition G of M into closed n-manifolds with $N_k \in G$ $(k=1,2)$. We also consider a closely related question: Given two closed n-manifolds N_1 and N_2, when does there exist a laminated cobordism (M, N_1, N_2)?

Homological equivalence of N_1 and N_2 is a necessary condition for the existence of a laminated cobordism (M, N_1, N_2); in his initial work [7] Daverman proved that then $H_*(M, N_k) = 0$ $(k=1,2)$. We show it not sufficient by presenting an example (Example 3.1) of a cobordism (M, N_1, N_2) satisfying this homology condition and such that there is no laminated cobordism (M', N_1, N_2).

On the other hand, a well-known sufficient condition for the existence of a lamination $(n \neq 3)$ is that (M, N_1, N_2) be an h-cobordism (each inclusion $i_k: N_k \to M$ is a homotopy equivalence), since then $M - N_2$ is homeomorphic to $N_1 \times [0,1)$. Other types of laminations exist, however; in the presence of wildness the decomposition elements can have varying homotopy types [7, Example 5.3]. Our chief interest centers on cobordisms (M, N_1, N_2) for which $i_2: N_2 \to M$ is a homotopy equivalence but $i_1: N_1 \to M$ is not. Under this assumption on i_2, it is easy to verify that $H_*(M, N_1) = 0$ and that the kernel of $i_{1\#}: \pi_1(N_1) \to \pi_1(M)$ is perfect. If, in addition, kernel$(i_{1\#})$ is the normal closure of a finitely generated perfect group, then as our main result we demonstrate how to impose a lamination on (M, N_1, N_2); in particular, we obtain M, up to attachment of a h-cobordism, as the mapping cylinder of an acyclic map from N_1 to an n-manifold homotopy equivalent to N_2 (Theorem 5.2).

2. Technical lemmas. This section provides a listing of some utilitarian facts about the manifolds admitting laminations.

DEFINITION 2.1. A *laminated cobordism* is a cobordism (M, N_1, N_2), where M is a compact $(n+1)$-manifold having boundary components N_1 and N_2, together with an usc decomposition G of M into closed n-manifolds such that $N_1, N_2 \in G$.

First we state two results from previous work.

LEMMA 2.2 [7, Corollary 6.3]. *In a laminated cobordism (M, N_1, N_2) the inclusion-induced $H_*(g) \to H_*(M)$ is an isomorphism for each $g \in G$.*

Received April 2, 1985. Final revision received February 17, 1986.
Michigan Math. J. 33 (1986).