AN EXPLICIT KOPPELMAN TYPE INTEGRAL FORMULA ON ANALYTIC VARIETIES

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Introduction. The purpose of this paper is to prove an explicit Koppelman type integral formula on analytic varieties, thus generalizing the Koppelman integral formula on domains in $\mathbf{C}^n$ as contained in Øvrelid [2]. This generalization is the content of Theorem 1. Roughly speaking, if V is an analytic variety in $\mathbf{C}^n$ defined by m holomorphic functions and M is domain on V, then we construct explicit kernels K_q so that for every $(0,q)$-form u with $\mathbf{C}^1$-coefficients on $\bar{M}$ we have:

$$u=\int_{\partial M} u\wedge K_q-\int_M \bar{\partial}u\wedge K_q-\bar{\partial}\left(\int_M u\wedge K_{q-1}\right).$$

The variety is assumed to have no singular point on $\bar{M}$. We also assume that ∂M, the boundary of M, is smooth. (See the next paragraph for a precise description of the setting and statement of the result.)

The construction of the kernels K_q and the proof of the integral formula are based on some results from [1]. These results generalized earlier results of Stout [4]. We also follow ideas from Øvrelid [2]. We will use the standard notation and terminology for differential forms (see, e.g., Rudin [3] and Wells [5]; see also Øvrelid [2]).

Description of the setting. Let $D\subset\mathbf{C}^n$ be a bounded domain with smooth boundary and let $\gamma_j(\zeta,z)$, $j=1,\dots,n$, be smooth functions defined for $\zeta\in\bar{D}$, $z\in D$ such that:

(i) $(\zeta-z,\gamma(\zeta,z))=:\sum_{j=1}^n(\zeta_j-z_j)\gamma_j(\zeta,z)\neq 0$ for $\zeta\neq z$;
(ii) $\gamma_j(\zeta,z)=\bar{\zeta}_j-\bar{z}_j$ for $|\zeta-z|<$ small constant.

Let $h_1,\dots,h_m$ be m, $m<n$, holomorphic functions in a domain Ω with $\Omega\supset\bar{D}$. Let $h_{ij}(\zeta,z)$ be holomorphic functions (in $(\zeta,z)\in\Omega\times\Omega$) so that

$$h_i(\zeta)-h_i(z)=\sum_{j=1}^n h_{ij}(\zeta,z)(\zeta_j-z_j),\quad i=1,\dots,m,\ (\zeta,z)\in\Omega\times\Omega.$$

Set $V=:\{z\in\Omega: h_1(z)=\cdots=h_m(z)=0\}$ and set $M=:V\cap D$ and $\partial M=:V\cap(\partial D)$. Define

$$|\nabla(h_1,\dots,h_m)(\zeta)|^2=:\sum_{1\le j_1<\cdots<j_m\le n}\left|\frac{\partial(h_1,\dots,h_m)}{\partial(\zeta_{j_1},\dots,\zeta_{j_m})}(\zeta)\right|^2.$$

Our assumptions are that $|\nabla(h_1,\dots,h_m)|\neq 0$ on $\bar{M}$, that is, that variety V has no singular point on $\bar{M}$ and that V meets ∂D transversally. Thus M is a complex manifold of (real-) dimension $2n-2m$ and ∂M is a smooth manifold of dimension $2n-2m-1$.

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