TAUBERIAN THEOREMS FOR PLURIHARMONIC FUNCTIONS WHICH ARE BMO OR BLOCH

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0. Introduction. Suppose f is a bounded pluriharmonic function in the unit ball of $\mathbf{C}^n$. It is a corollary to Theorem 3 of [5] that f has a radial limit at a given boundary point if and only if the (a.e.) boundary values of f have a certain "derivative" at that point. The main result of the present paper is an analogous result for pluriharmonic functions satisfying a Bloch condition: see Theorem 1 below. Note that since Bloch functions need not have radial limits a.e., the statement of Theorem 1 involves instead certain linear functionals on the Bloch space which reduce to the average of the boundary values over certain sets, *if* these boundary values exist. Thus if f is Bloch and equals the Poisson–Szegö integral of a measure, the existence of a radial limit is equivalent to the existence of a "derivative" of the boundary measure (Corollary 1). In particular, in case f is both pluriharmonic and the Poisson–Szegö integral of a BMO function, we obtain Corollary 2. (The present Corollary 2 was the main result in the original version of this paper. Peter Jones, in collaboration with Carl Sundberg, suggested that exactly the same proof would yield Corollary 1, a stronger result.)

Theorem 1 will follow from Theorem 2, concerning Bloch functions in the unit *disc*. The averages in Theorem 2 are taken over open subsets of the disc, so that the non-existence of boundary values is no longer a problem. This reduction from a subset of the boundary of the unit ball in $\mathbf{C}^n$ to an open subset of $\mathbf{C}$ is available only if $n \geq 2$; this is the reason for the hypothesis "$n \geq 2$" in Theorem 1. (The statement of Theorem 1 is still true for $n=1$, but the proof is very much different and will appear elsewhere. Note that the case $n=1$ of Corollary 2 is contained in [6].)

Theorem 2, in turn, will follow from Theorem 3, which may be regarded as a quantitative version of results implicit in [5]; Theorem 3 is possibly of some interest in itself.

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1. Statement of results. Let $n \geq 2$. Let B denote the unit ball of $\mathbf{C}^n$, $S = \partial B$; let σ denote the rotation-invariant probability measure on S. Let $\mathcal{B} = \mathcal{B}(B)$ be the Bloch space, the space of all *pluriharmonic* functions $f: B \to \mathbf{C}$ such that the quantity

$$\frac{1-|z|^2}{n+1} \sum_{i,j=1}^{n} (\delta_{i,j} - z_i \bar{z}_j)\left(\frac{\partial f}{\partial z_i}\frac{\partial \bar{f}}{\partial \bar{z}_j} + \frac{\partial \bar{f}}{\partial z_i}\frac{\partial f}{\partial \bar{z}_j}\right)$$

is bounded in B. (This is simply the square of the norm on covectors dual to the Bergman metric, applied to the gradient of f. Various other characterizations of

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