SOME SEIFERT FIBER SPACES WHICH ARE BOUNDARIES

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Hamrick and Royster [6] proved that all compact flat manifolds are boundaries. Their argument uses a refined version of a theorem of R. Stong together with M. Gordon's result [5]. Recently this was generalized to some almost flat manifolds by Farrell and Zdravkovska [4].

In this article we shall prove certain Seifert fiber spaces are boundaries. This class of Seifert fiber spaces contains all flat manifolds, those almost flat manifolds covered by the first part of [4], and more importantly, all the manifolds of non-positive sectional curvature satisfying Assumption B below.

We formulate the generalization of Stong's theorem [6] in a more general setting so that we can apply it to Seifert fiber spaces. Then we use the same argument as in [6] and [4]. An important point is that the Seifert fiber structure gives rise to a free $(\mathbf{Z}_2)^n$ action on a finite covering of the manifold.

In this paper, a Seifert fibering $M \to B$ will mean a smooth closed manifold M with an injective Seifert fiber structure where the typical fiber is a flat torus T^n. More precisely, M is a smooth closed manifold such that

(i) $\pi_1 M$ contains a normal subgroup $\mathbf{Z}^n$ $(n>0)$,

(ii) there exists $\mathbf{R}^n \subset \mathrm{Diffeo}(\tilde{M})$ containing $\mathbf{Z}^n$ as a uniform lattice, and

(iii) $\mathbf{R}^n$ is normalized by $\pi_1 M$.

Of course, we are considering $\pi_1 M$ as a subgroup of $\mathrm{Diffeo}(\tilde{M})$ where $\tilde{M}$ is the universal covering of M. Generally, the $\mathbf{R}^n$-action on $\tilde{M}$ does not yield a torus action on M, but the universal covering $\tilde{M}$ splits as a direct product $\mathbf{R}^n \times W$, where W is a simply connected smooth manifold on which $Q = \pi_1 M/\mathbf{Z}^n$ acts properly discontinuously with compact quotient. Thus, $B = Q\backslash W$ and

$$M = \pi_1 M\backslash(\mathbf{R}^n \times W) = Q\backslash(T^n \times W).$$

Note that even though Q acts on $T^n \times W$ as a group of covering transformations, it does not act freely on the W-factor. In general, the base space B is an orbifold where the fibers over regular (= unbranched) points of B are called typical. Otherwise, they are called singular. Singular fibers are finitely covered by the typical fiber T^n and are flat Riemannian manifolds.

We shall denote $\pi_1 M$ simply by π. $C_\pi(\mathbf{Z}^n)$ denotes the centralizer of $\mathbf{Z}^n$ in π. We make two assumptions as follows.

ASSUMPTION A. $C_\pi(\mathbf{Z}^n)$ has finite index in π.

ASSUMPTION B. $C_\pi(\mathbf{Z}^n)/\mathbf{Z}^n$ has no 2-torsion.

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