REPRESENTATIONS OF THE MAUTNER GROUP AND COCYCLES OF AN IRRATIONAL ROTATION

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1. Introduction. The five-dimensional Lie group known as the Mautner group is the smallest connected Lie group whose unitary dual remains unknown. Its representation theory is linked to the cohomology of functions on the circle under an irrational rotation. In this paper, we use cocycles to produce a five parameter family of representations of the Mautner group. This family completes all the known families in a natural way and extends them to reveal the dependence of the representation theory of the group on the angle defining it.

The Mautner group M is ordinarily defined to be the set $\mathbf{C}\times\mathbf{C}\times\mathbf{R}$ together with the multiplication rule

$$(z,w,t)(z',w',t')=(z+e(t/2\pi)z',w+e(t)w',t+t'),$$

where $e(x)=e^{2\pi ix}$; that is, M is the semidirect product of two-dimensional complex space with the real line, where the real number t acts on $\mathbf{C}^2$ by the matrix

$$\begin{bmatrix} e(t/2\pi) & 0 \\ 0 & e(t) \end{bmatrix}.$$

If α and β are nonzero real numbers, a similar semidirect product $M_{\alpha,\beta}$ can be defined, where the real number t acts this time by the matrix

$$\begin{bmatrix} e(\alpha t) & 0 \\ 0 & e(\beta t) \end{bmatrix}.$$

If the quotient $\theta=\beta/\alpha$ is irrational then $M_{\alpha,\beta}$ exhibits the "Winding Line" phenomenon in its structure, so that all these groups seem to be analogous from the point of view of ergodic theory. In addition, the primitive ideal spaces of the $M_{\alpha,\beta}$'s, for β/α irrational, are all identical: $\mathrm{Prim}(M_{\alpha,\beta})$ is the union of the real line (characters) with the set of nondegenerate tori $S_\rho\times S_r$ in $\mathbf{C}^2$ (normal factors).

It would be reasonable to expect the representation theory of all these groups to be alike as well. For any irrational β/α, it is known that $M_{\alpha,\beta}$ is not of type I, so that its unitary dual (equivalence classes of irreducible unitary representations) cannot be parameterized in a smooth way. Further, the known parameterized families of representations of the ordinary Mautner group (see [2], [6], [3]) can be easily transferred to an arbitrary $M_{\alpha,\beta}$. However, we have found by extending these families that the representation theory of $M_{\alpha,\beta}$ depends substantially on the number theoretic properties of the quotient β/α. We present here a five- (real) parameter family of formulae defining irreducible unitary representations of the group $M_{\alpha,\beta}$ for all α and β with β/α irrational. The unitary

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