GROUPS OF AUTOMORPHISMS OF HYPERELLIPTIC KLEIN SURFACES OF GENUS THREE

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Dedicated to Prof. E. Linés on the occasion of his academic jubilee

1. Introduction. The problem of determining the group of automorphisms of a surface is a classical one that goes back to Hurwitz [15]. The study of the groups of automorphisms of Klein surfaces has grown in the last ten years. Our goal in this paper is to determine the group of automorphisms of each hyperelliptic Klein surface of genus 3. The same question was solved for genus 1 in [1] and for genus 2 in [9].

In Riemann surfaces these questions were first studied by Wiman [31]. He solves the problem for genus 2. More recently some results about hyperelliptic Riemann surfaces of genus 3 have been obtained by A. and I. Kuribayashi [16, 17, 18].

The techniques used in this paper, involving NEC groups, are different from those of the Riemann case.

We now describe the contents of our paper. In §2 we introduce the terminology about Klein surfaces and NEC groups, and establish a technical result.

Section 3 is devoted to the study of some properties of the automorphisms of Klein surfaces of genus three, and in §4 we introduce the method for checking when a group of automorphisms of an arbitrary bordered compact Klein surface is the full group.

In §5 we obtain the main result. All the groups that are the full group of automorphisms of each hyperelliptic Klein surface of genus 3 are classified according to the topological type of the surface.

From the well-known functorial equivalence established by Alling and Greenleaf [2] between the category of real irreducible algebraic curves and the one of bordered Klein surfaces, and our results [8] about the relation between hyperelliptic Klein surfaces and hyperelliptic real algebraic curves, we will translate in §6 the results obtained for surfaces in §5 to the language of curves.

2. NEC groups and Klein surfaces. Klein surfaces, introduced from a modern point of view by Alling and Greenleaf [2], are studied by means of NEC groups since the results of Preston and May.

If X is a Klein surface of algebraic genus $p \geq 2$, it may be expressed as D/Γ, where $D = \{z \in \mathbf{C} \mid \mathrm{Im}(z) \geq 0\}$ and Γ is an NEC group (see Preston [25]).

NEC groups, introduced by Wilkie [30], are discrete subgroups of the group G of isometries of the hyperbolic plane, with compact quotient space.

Received April 10, 1984. Revision received December 3, 1984.

The authors were partially supported by "Comisión Asesora de Investigación Científica y Técnica".

Michigan Math. J. 33 (1986).