ON SUBMANIFOLDS WITH PLANAR NORMAL SECTIONS

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1. Introduction. Let M be a submanifold of dimension n in a Euclidean m-space E^m. For any point p in M and any unit vector t at p tangent to M, the vector t and the normal space N_pM of M at p determine an $(m-n+1)$-dimensional vector subspace $E(p,t)$ of E^m through p. The intersection of M and $E(p,t)$ gives rise to a curve γ in a neighborhood of p which is called the normal section of M at p in the direction t. The submanifold M is said to have planar normal sections if normal sections of M are planar curves. In this case, for any normal section γ, we have $\gamma' \wedge \gamma'' \wedge \gamma''' = 0$. A submanifold M is said to have pointwise planar normal sections if, for each p in M, every normal section γ at p satisfies $\gamma' \wedge \gamma'' \wedge \gamma''' = 0$ at p. Submanifolds with (pointwise) planar normal sections were investigated in [1, 2, 3, 6]. B. Y. Chen [3] classified surfaces in E^m with planar normal sections, and he proved the following theorem:

THEOREM A. *Let M be a surface in E^m with planar normal sections. If, locally, M does not lie in a 3-dimensional hyperplane of E^m, then M is an open subset of a Veronese surface in a 5-dimensional hyperplane of E^m.*

In the following, by a Veronese submanifold V^n we mean a real projective n-space isometrically imbedded in $E^{n+n(n+1)/2}$ by its first standard imbedding (cf. [4, pp. 141–148]).

In this paper, we generalize Theorem A to higher dimensions. We shall prove the following theorems.

THEOREM B. *Let M be an n-dimensional submanifold in E^m with planar normal sections. If, locally, M does not lie in an $(n(n+1)/2)$-dimensional affine subspace of E^m, then M is an open portion of a Veronese submanifold V^n in an $(n+n(n+1)/2)$-subspace of E^m.*

THEOREM C. *Let M be a 3-dimensional submanifold in E^m with planar normal sections. If, locally, M does not lie in a 5-space E^5 of E^m, then M is an open portion of a Veronese submanifold V^3 in E^9 or is the Riemannian direct product of the real line* **R** *with the Veronese surface.*

2. Proof of Theorem B. Let M be a submanifold in E^m, ∇ and $\tilde{\nabla}$ be the covariant derivatives of M and E^m, respectively. For any two vector fields X, Y tangent to M, the second fundamental form h is given by $h(X,Y) = \tilde{\nabla}_X Y - \nabla_X Y$. For any vector field ξ normal to M, we have $\tilde{\nabla}_X \xi = -A_\xi X + \nabla_X^\perp \xi$, where A_ξ is the Weingarten map associated with ξ and $\nabla^\perp$ is the normal connection of the normal bundle $N(M)$. Define the covariant derivative of h by

(2.1) $$(Dh)(X,Y,Z) = \nabla_X^\perp(h(Y,Z)) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z),$$

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