RANDOM SERIES AND BOUNDED MEAN OSCILLATION

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Dedicated to George Piranian

A function f analytic in the unit disk $\mathbf{D}$ is said to belong to the *Hardy space* H^p, $0<p<\infty$, if its integral means

$$M_p(r,f)=\left\{\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^p\,d\theta\right\}^{1/p}$$

remain bounded as r tends to 1. We let $M_\infty(r,f)$ denote the maximum of $|f(z)|$ on the circle $|z|=r<1$. Thus H^∞ is the class of bounded analytic functions in $\mathbf{D}$. A function $f\in H^1$ is said to be in the space BMOA if its boundary function $F(t)=f(e^{it})$ is of *bounded mean oscillation*:

$$\sup_I \frac{1}{|I|}\int_I |F(t)-F_I|\,dt<\infty,$$

where $|I|$ is the length of the interval I and

$$F_I=\frac{1}{|I|}\int_I F(t)\,dt.$$

The *Bloch space* $\mathcal{B}$ consists of all analytic functions f for which

$$\sup_{z\in\mathbf{D}}(1-|z|)|f'(z)|<\infty.$$

The proper inclusions

$$H^\infty\subset \text{BMOA}\subset \mathcal{H}^*=\bigcap_{p<\infty}H^p$$

and BMOA $\subset\mathcal{B}$ are well known. Moreover,

$$\mathcal{B}\not\subset\mathcal{H}_*=\bigcup_{p>0}H^p.$$

A useful criterion for an analytic function f to belong to BMOA is that

$$d\mu(z)=(1-|z|)|f'(z)|^2\,dx\,dy$$

be a *Carleson measure* on $\mathbf{D}$. See [2] and [3] for further background.

We shall be concerned with *random power series*

$$f(z)=\sum_{n=0}^{\infty}\epsilon_n a_n z^n,\quad \epsilon_n=\pm 1,$$

where the ϵ_n are random signs and $\limsup\sqrt[n]{|a_n|}\le 1$. More precisely, such functions have the form

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