ON THE UNITARY EQUIVALENCE OF CLOSE C^*-ALGEBRAS

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Introduction. A central question in the theory of perturbations of C^*-algebras is to determine which C^*-algebras A satisfy the following property: Every C^*-algebra B "sufficiently close" to A is unitarily equivalent to it (cf. [3], [4], [11]). In this paper we use "Ext" theory in order to find new C^*-algebras with this property.

Let D be a separable C^*-subalgebra of a C^*-algebra C and suppose that D is an extension of a C^*-algebra A by a C^*-algebra I. Under certain assumptions on A and I we show that if D' is a C^*-subalgebra of C, "sufficiently close" to D, then D and D' are unitarily equivalent. To that end, we prove that D' is also an extension of A by I and show that these two extensions are unitarily equivalent. This second problem is dealt with by viewing the two extensions of A by I given by D and D' through the six-term exact sequence of K-theory associated with the two extensions, using the Rosenberg and Schochet universal coefficient formula (cf. [13]) and Theorem 2.11 of [9].

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NOTATIONS. Throughout this paper H will denote a separable infinite dimensional Hilbert space. $\mathcal{L}(H)$ is the C^*-algebra of bounded linear operators on H and $K(H)$ is the C^*-algebra of compact operators. If A is a C^*-algebra $M(A)$ denotes the multiplier algebra of A.

The distance between the C^*-subalgebras A and B of a C^*-algebra C is defined by

$$d(A,B)=\operatorname{Max}\left\{\sup_{a\in A_1}\inf_{b\in B_1}\|a-b\|;\ \sup_{b\in B_1}\inf_{a\in A_1}\|a-b\|\right\},$$

where A_1 and B_1 denote the unit balls of A and B respectively.

1. Some results from the theory of extensions. Here we recall some facts about Kasparov's bi-functor $\operatorname{Ext}(A,B)$ (cf. [8]).

1.1. Let A be a separable nuclear C^*-algebra and B a C^*-algebra with countable approximate unit. An (A,B) extension is a short exact sequence

$$0\to B\otimes K(H)\to D\xrightarrow{\phi} A\to 0.$$

Such an extension will be denoted by the pair (D,ϕ). We note that (cf. [2]) such extensions are in one-to-one correspondence with *-homomorphisms $\sigma: A\to M(B\otimes K(H))/B\otimes K(H)$. Two extensions σ_1 and σ_2 are said to be unitarily equivalent (write $\sigma_1 \underset{u}{\sim} \sigma_2$) if there exists a unitary $u\in M(B\otimes K(H))$ such

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