MULTILINEAR CONVOLUTIONS AND TRANSFERENCE

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1. Introduction. In this paper, we present some theorems which are basic to the study of a wide class of nonlinear operators which arise in partial differential equations and many other parts of analysis. These results are part of a continuing program of nonlinear analysis developed over the past several years by R. R. Coifman, A. McIntosh, Y. Meyer, and others (see, for example, [1], [2], and [3]). A typical problem is that of obtaining L^p estimates for linear operators $T(b)$ which depend nonlinearly on a functional parameter b in a Banach space B. By analogy to the calculus of functions in finite dimensions, T is said to be *analytic at the origin in B* if and only if for all b with $\|b\|_B$ sufficiently small and for all $f \in L^p(X)$,

$$T(b)f = \sum_{k=0}^{\infty} M_{k+1}(b, \dots, b, f), \tag{1.1}$$

where M_{k+1} is a bounded $(k+1)$-multilinear operator on $(B)^k \times L^p$ which satisfies an estimate of the form

$$\|M_{k+1}(b, \dots, b, f)\|_p \le C^k \|b\|_B^k \|f\|_p \tag{1.2}$$

for some absolute constant $C > 0$. The multilinear operator $k!\, M_{k+1}(b, \dots, b, f)$ is, in fact, the action of f of the kth Fréchet differential of T at 0 in the direction b. In order to prove that T depends analytically on b, it suffices to find an explicit representation for T as a convergent "power series" of multilinear operators in a neighborhood of the origin in B. The problem of obtaining L^p estimates for T is thereby reduced to that of obtaining L^p estimates for the Taylor coefficients of T. Thus we are led to investigate certain broad classes of multilinear operators which arise naturally as the Fréchet differentials of nonlinear operators.

Of particular interest are the multilinear convolutions: multilinear operators which commute with the simultaneous action of a group of measure-preserving transformations of the underlying measure space. Specifically, let (X, μ) be a σ-finite measure space, and let $\{U_t\}$ be a group of measure-preserving transformations of X, indexed by $\mathbf{R}^n$. Let B be a Banach function space on X, and suppose that for $b \in B$, $\|b \circ U_t\|_B = \|b\|_B$; that is, the norm on B is invariant under the action of $\{U_t\}$. A k-multilinear operator M_k is called a *multilinear convolution* if and only if

$$M_k(f_1 \circ U_t, \dots, f_k \circ U_t) = M_k(f_1, \dots, f_k) \circ U_t. \tag{1.3}$$

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