ANALYTIC MULTIPLIERS OF BERGMAN SPACES

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Basics and introduction. Let W be a nonempty region in the complex plane and let $L^p(W)$ be the usual Lebesgue p-space of complex functions with domain W, relative to the Lebesgue two-dimensional area measure dm. For $0<p\leq\infty$, let the Bergman p-space be defined by $L_a^p(W)=L^p(W)\cap H(W)$, where $H(W)$ is the space of analytic functions on W. For $f\in L_a^p(W)$ let

$$\|f\|_p=\left(\int_W |f|^p\,dm\right)^{1/p} \quad \text{if } 0<p<\infty$$

$$=\sup_{z\in W}|f(z)| \qquad \text{if } p=\infty.$$

The class $L_a^\infty(W)$ of bounded analytic functions on W is usually denoted by $H^\infty(W)$. Let $0<p\leq\infty$ and let $\{f_n\}$ be a Cauchy sequence in $L_a^p(W)$. Then by using a theorem of Hardy and Littlewood ([8], Chapter 3, Lemma 3.7), one deduces the existence of f in $H(W)$ such that $f_n\to f$ uniformly on compact sets. It follows that if $p\geq 1$ then $L_a^p(W)$ is a Banach space, and that if $0<p<1$ then $L_a^p(W)$ is an F-space.

$L_a^2(W)$ is a Hilbert space, with the inner product $\langle f,g\rangle=\int_W f\bar{g}\,dm$. For each $w\in W$ there exists a unique k_w in $L_a^2(W)$ such that $f(w)=\int_W f\bar{k}_w\,dm$ for each f in $L_a^2(W)$. This k_w is called the reproducing kernel associated with w. Let D denote the unit disc. When $W=D$, we have

$$k_w(z)=\frac{1}{\pi}\cdot\frac{1}{(1-\bar{w}z)^2}$$

for $z\in D$ and $w\in D$. Let P be the orthogonal projection from $L^2(W)$ onto $L_a^2(W)$, so that

$$P(f)(w)=\int_W f\bar{k}_w\,dm.$$

Taking this as the definition of $P(f)$ for each f in $L^p(D)$, Zaharjuta and Judovic [16] (also see [4]) proved that P projects $L^p(D)$ onto $L_a^p(D)$ continuously for $1<p<\infty$. An immediate consequence would be that the dual of $L_a^p(D)$ can be identified with $L_a^q(D)$, where $1<p<\infty$ and $1/p+1/q=1$.

The map P does not project $L^1(D)$ to $L_a^1(D)$ continuously. However $L^1(D)$ can be continuously projected onto $L_a^1(D)$ ([3]). In fact, it is not hard to see that

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