ON THE STRONG SUMMABILITY OF DOUBLE ORTHOGONAL SERIES

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1. Introduction. Let (X, F, μ) be an arbitrary positive measure space and let $\{\varphi_{ik}(x): i, k=1,2,\dots\}$ be an orthonormal system on X. We consider the double orthogonal series

$$\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} a_{ik} \varphi_{ik}(x), \tag{1.1}$$

where $\{a_{ik}: i, k=1,2,\dots\}$ is a sequence of coefficients for which

$$\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} a_{ik}^2 < \infty. \tag{1.2}$$

Then, by the well-known Riesz-Fischer theorem, there exists a function $f(x) \in L^2 = L^2(X, F, \mu)$ such that the rectangular partial sums

$$s_{mn}(x) = \sum_{i=1}^{m} \sum_{k=1}^{n} a_{ik} \varphi_{ik}(x) \quad (m, n=1,2,\dots)$$

of series (1.1) converge to $f(x)$ in the L^2-metric:

$$\int [s_{mn}(x) - f(x)]^2 \, d\mu(x) \to 0 \quad \text{as } \min(m, n) \to \infty.$$

Here and in the sequel, the integrals are taken over the whole space X.

It is a basic fact that condition (1.2) itself does not ensure the pointwise convergence of $s_{mn}(x)$ to $f(x)$. The extension of the famous Rademacher-Menšov theorem proved by a number of authors (see e.g. [1], [7], etc.) reads as follows.

THEOREM A. *If*

$$\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} a_{ik}^2 [\log(i+1)]^2 [\log(k+1)]^2 < \infty,$$

then

$$s_{mn}(x) \to f(x) \quad \textit{a.e. as } \min(m, n) \to \infty$$

and there exists a function $F(x) \in L^2$ such that

$$\sup_{m, n \geq 1} |s_{mn}(x)| \leq F(x) \quad \textit{a.e.}$$

In this paper all logarithms are to the base 2.

The next theorem (see e.g. [8]) gives information on the order of magnitude of $s_{mn}(x)$ in the more general setting of (1.2).

THEOREM B. *Under condition* (1.2),

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