ON THE EXTENSION OF SEPARATELY HYPERHARMONIC FUNCTIONS AND H^p-FUNCTIONS

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1. Introduction.

1.1. Grauert and Remmert [7, p. 175] proved that in $\mathbf{C}^n$ analytic sets are removable singularities for plurihyperharmonic functions which are locally bounded below. In fact, their result was extended also for complex spaces [7, Satz 3, p. 181]. Lelong [12, Theorem 2, p. 279] (see also [14, Theorem 4, p. 35] or [8, Theorem 1.2 (b), p. 704]) extended the classical result by showing that in $\mathbf{C}^n$ ($\mathbf{R}^{2n}$-) polar sets are removable in this situation. Using Hausdorff measure, Shiffman [21, Theorem 3, p. 338] gave other extension results for plurihyperharmonic functions.

In Theorem 4.1 below we give a similar result to Lelong's result for functions which are separately hyperharmonic with respect to each complex variable. In fact, we allow our exceptional sets to be slightly larger than polar sets. For this purpose in Section 2 we define n-small sets in $\mathbf{C}^n$. Since polar sets are n-small, also n-polar and sets of finite $(2n-2)$-dimensional Hausdorff measure are n-small. In addition, our sets include n-negligible sets. Note that there are (at least nonmeasurable) n-negligible sets which are not polar (see Remark 2.8 below). For the definition of n-polar and n-negligible sets see [5, Definition 3.10, p. 246], [11, p. 597], [24, Definitions 3.1 and 3.2, p. 32], and [4, p. 284]. In Section 4 we give also some other extension results for functions which are separately hyperharmonic with respect to each complex variable.

In Section 5 we then apply Theorem 4.1 to get extension results for H^p-functions in $\mathbf{C}^n$. Our result, Theorem 5.2 below, includes the results of [11, Theorem 2, p. 597], [4, Remark 3, p. 286], [5, Theorem B, p. 241], and [17, Theorem 3.5, p. 287].

However, before giving the above results we begin in Section 3 with a remark concerning separately hyperharmonic functions. Avanissian [3, Theorem 9, p. 140] (see also [10, Theorem, p. 31]) proved that a separately hyperharmonic function is hyperharmonic if it is locally bounded below. Using a different method, Arsove [2, Theorem 2, p. 622] showed that it is enough that the function has locally an integrable minorant. In Theorem 3.4 below we point out that Arsove's result can also be obtained directly and shortly from Avanissian's result.

For the properties of distributions see [20]. For the properties of hyperharmonic and holomorphic functions see [10], [9], [14] and [19].

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1.2. In addition to the standard notation we use the following, which is partly similar to the notation used in [10].

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