EXTRINSIC SPHERES IN A KÄHLER MANIFOLD

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1. Introduction. An $n(\geqslant 2)$-dimensional submanifold of an arbitrary Riemannian manifold is called an extrinsic sphere if it is totally umbilical and has nonzero parallel mean curvature vector [5]. An n-dimensional Riemannian manifold is called an intrinsic sphere if it is locally isometric to an ordinary sphere in a Euclidean space. Since extrinsic spheres are natural analogues of ordinary spheres in a Euclidean space from the extrinsic point of view, it is natural to ask when an extrinsic sphere is to be an intrinsic sphere. In this situation, it is well known that an extrinsic sphere in a Euclidean space is an intrinsic sphere. However, in general, an extrinsic sphere is not always an intrinsic sphere (see [4: p. 66], for example). On the other hand, when the ambient manifold is a Kähler manifold, B. Y. Chen has proved the following Theorem A:

THEOREM A [2]. *A complete, connected, simply connected and even-dimensional extrinsic sphere of a Kähler manifold is isometric to an ordinary sphere if its normal connection is flat.*

He has also given counterexamples which are not isometric to an ordinary sphere in odd-dimensional case [3].

In this paper, we shall try to classify a complete, connected and simply connected extrinsic sphere of a Kähler manifold. That is, we shall prove the following Theorem:

THEOREM. *A complete, connected and simply connected extrinsic sphere M^n in a Kähler manifold $\tilde{M}^{2m}$ is one of the following:*

(1) *M^n is isometric to an ordinary sphere,*

(2) *M^n is homothetic to a Sasakian manifold,*

(3) *M^n is a totally real submanifold and the f-structure is not parallel in the normal bundle.*

Here we note that case (2) *and* (3) *occur only when $n=$odd and $m\geqslant n+1$, respectively.*

2. Preliminaries. Let $\tilde{M}$ be a Riemannian manifold of dimension m and M an n-dimensional submanifold of $\tilde{M}$. Let $\langle\ ,\ \rangle$ be the metric tensor field on $\tilde{M}$ as well as the induced metric on M. We denote by $\tilde{\nabla}$ the covariant differentiation in $\tilde{M}$ and by ∇ the covariant differentiation in M determined by the induced metric on M. Then the Gauss-Weingarten formulas are given by

$$\tilde{\nabla}_X Y=\nabla_X Y+h(X,Y),$$

$$\tilde{\nabla}_X N=-A_N X+\nabla_X^{\perp} N,$$

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