INCOMPRESSIBILITY OF SURFACES AFTER DEHN SURGERY

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Introduction. This paper was, at first, motivated by Hatcher and Thurston's question [4] of whether each Dehn surgery on a two-bridge knot along the boundary of an incompressible and ∂-incompressible surface leads to a Haken manifold. However, a deeper motivation was that our investigation was intended as a first step to approaching the Waldhausen Conjecture that each P^2-irreducible manifold with infinite fundamental group has a finite sheeted covering which is a Haken manifold [13].

First we prove Theorem 1.4, which says that incompressibility of a surface is preserved after Dehn surgery for an unknotted surface with one boundary component and with some extra conditions with two boundary components. Then we show that the condition that a surface is unknotted cannot be dropped, and the assumption about the number of the boundary components of the surface is essential too (Example 1.14). We use Theorem 1.4 to answer the Hatcher–Thurston question. Later we solve the similar problem for punctured torus bundles over S^1. This allows us to construct a large class of non-Haken, non-Seifert but almost Haken manifolds (Propositions 3.1–3.3). We consider also branched coverings of some non-Haken manifolds (Proposition 3.4).

Finally we show that the assumptions of Theorem 1.4 are often satisfied and Theorem 1.4 has several applications. Using Jaco's theorem about hierarchies [6], we find a condition for 3-manifolds which is sufficient to get unknotted surfaces (Proposition 4.3 and Corollary 4.5), and then we give examples of manifolds satisfying this condition (some closed 3-braids).

We end the paper by the remark that Theorem 1.4 can be used to prove property R for a huge class of knots.

1. Main theorem. We work in the PL-category.

DEFINITION 1.1. (a) Let M be a 3-manifold and F a surface which is either properly embedded in M or contained in ∂M. We say that F is *compressible in M* if one of the following conditions is satisfied:

(i) F is a 2-sphere which bounds a 3-cell in M, or

(ii) F is a 2-cell and either $F \subset \partial M$ or there is a 3-cell $X \subset M$ with $\partial X \subset F \cup \partial M$, or

(iii) there is a 2-cell $D \subset M$ with $D \cap F = \partial D$ and with ∂D not contractible in F.

We say that F is *incompressible* if it is not compressible.

(b) Let F be a submanifold of a manifold M. We say that F is *π_1-injective in M* if the inclusion-induced homomorphism from $\pi_1(F)$ to $\pi_1(M)$ is an injection.

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