WEIGHTED L^p ESTIMATES FOR THE CAUCHY INTEGRAL OPERATOR

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Introduction and statement of basic result. In 1977 A. P. Calderón proved that the Cauchy Integral Operator for a curve $(x, A(x))$ is bounded on L^2 provided that A' is in L^∞ with sufficiently small L^∞ norm. Four years later R. R. Coifman, Y. Meyer, A. McIntosh, and G. David developed new techniques and were able to remove the restriction on the size of the L^∞ norm of A'. Furthermore, as a result of the almost everywhere existence of the Cauchy Integral for rectifiable curves one can deduce the existence of a weighted L^2 estimate for such curves (see [2]). The main objective of this paper is the direct derivation of weighted L^p estimates for the Cauchy Integral Operator with weights that can be explicitly exhibited in a way that clarifies the role played by the geometry of the curve. We will prove the following:

THEOREM A. *There exist constants k_1 and k_2 such that for all $p>1$ there exists a constant C_p for which the following inequality holds:*

$$\int C_*^p(A,f)(x)\frac{dx}{(((1+S_q(A'))^{k_1})^*)^{k_2}} \leqslant C_p \int |f(t)|^p\,dt$$

where $C_*(A,f) = \sup_{\epsilon>0}|C_\epsilon(A,f)|$ and $C_\epsilon(A,f)(x)$ is the truncated operator corresponding to the Cauchy Singular Integral Operator

$$C(A,f)(x) = \text{p.v.}\int \frac{1+iA'(y)}{x-y+i(A(x)-A(y))} f(y)\,dy.$$

$S_q(A')$ denotes the q-sharp function of A':

$$S_q(A')(x) = \sup_Q\left(\frac{1}{|Q|}\int_Q |A'(y)-m_Q(A')|^q\,dy\right)^{1/q}$$

with $m_Q(.)$ denoting the mean over the specified interval, and the sup taken over all intervals containing x. $(.)^*$ denotes the Hardy-Littlewood Maximal Function and the variable t stands for arc-length.

The proof proceeds in three steps: (a) we use the Coifman-Meyer-McIntosh theorem (CMM) mentioned in the beginning as an a priori estimate to derive a provisional form of Theorem A via a good-λ inequality; (b) we use the measure theoretic and geometric techniques of step (a) again to show that the provisional result obtained there implies an improvement of itself, thus obtaining an L^p estimate; and (c) we prove a weak-type (1, 1) estimate which implies Theorem A by interpolation. The proof therefore contains a bootstrap argument from the CMM theorem following ideas used by G. David to derive the CMM theorem

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