ON COMPACT COMPLETELY BOUNDED MAPS OF C^*-ALGEBRAS

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1. Introduction. A C^*-algebra is said to be subhomogeneous if all its irreducible representations are on Hilbert spaces of dimension at most some positive integer. In the proof of [5: Theorem 3] we proved that if a C^*-algebra A is infinite dimensional and a C^*-algebra B is not subhomogeneous, then there exists a compact linear map of A into B which is not completely bounded and which is not a linear combination of positive linear maps.

In this paper we show that if ϕ is a map of a C^*-algebra into a nuclear C^*-algebra such that there exists a sequence of linear maps of finite rank converging to ϕ in the completely bounded norm, then ϕ is a linear combination of compact, completely positive maps. We also study, in the completely bounded norm, the closure of the set of linear maps of finite rank between some C^*-algebras. As an application of a result of Smith [8: Theorem 2.8], we prove that if on the algebraic tensor product $A \odot B$ of two C^*-algebras A and B, the greatest cross norm γ is equivalent to the projective C^*-cross norm, then either A or B is finite dimensional.

2. Preliminaries. For C^*-algebras A and B, let $B(A,B)$, $K(A,B)$ and $F(A,B)$ denote the set of bounded linear maps of A into B, the set of compact linear maps of A into B and the set of linear maps of finite rank of A into B, respectively.

Let ϕ be a map in $B(A,B)$. It is possible to define associated maps $\phi \otimes \mathrm{id}_n : A \otimes M_n \to B \otimes M_n$, and ϕ is said to be completely positive if each $\phi \otimes \mathrm{id}_n$ is positive, and completely bounded if $\sup_n \|\phi \otimes \mathrm{id}_n\| < \infty$. This quantity is called the completely bounded norm $\|\phi\|_{\mathrm{cb}}$ when it exists. If ϕ is completely positive, $\|\phi\|_{\mathrm{cb}} = \|\phi\|$. Let $CB(A,B)$ denote the set of completely bounded maps of A into B. For ϕ in $B(A,B)$, if there exist completely positive maps ϕ_i of A into B such that $\phi = \phi_1 - \phi_2 + i(\phi_3 - \phi_4)$, then ϕ is said to have a completely positive decomposition. If a net $\{\phi_\beta\}$ in $CB(A,B)$ converges to ϕ in the norm $\|.\|_{\mathrm{cb}}$, we write $\text{cb-lim}_\beta\, \phi_\beta = \phi$.

Let $A \otimes_\alpha B$ denote the completion of the algebraic tensor product $A \odot B$ under a norm α. In particular $A \otimes_{\max} B$ and $A \otimes_{\min} B$ mean the projective and injective C^*-tensor products, respectively [9: Chapter IV, Section 4].

A C^*-algebra A is said to be nuclear if for every C^*-algebra B the C^*-norm on the algebraic tensor product $A \odot \mathrm{B}$ is uniquely determined [2].

For the theory of C^*-algebras, we refer to the book of Takesaki [9].

3. Nuclear range algebras. The following lemma is perhaps known. For completeness we include the proof.

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