BOUNDARY BEHAVIOR OF PROPER HOLOMORPHIC MAPPINGS

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1. Introduction. It has recently been proved in [4] and in [6] that if $f: D_1 \to D_2$ is a proper holomorphic mapping between smooth bounded pseudoconvex domains in $\mathbf{C}^n$, and if the Bergman projection associated to D_1 is globally regular, then f extends smoothly to $\bar{D}_1$. The purpose of this note is to indicate how this result extends to the more general setting where D_1 and D_2 are relatively compact domains inside Stein manifolds.

If D is a relatively compact domain in a Stein manifold M, the space $L^2_{n,0}(D)$ is defined to be the set of $(n,0)$ forms ω such that

$$\|\omega\|^2 = (\sqrt{-1}\,)^{n^2} \int_D \omega \wedge \bar{\omega}$$

is finite. The space $L^2_{n,0}(D)$ is a Hilbert space with inner product given by

$$(\omega, \eta) = (\sqrt{-1}\,)^{n^2} \int_D \omega \wedge \bar{\eta}.$$

The Bergman projection P associated to D is the orthogonal projection of $L^2_{n,0}(D)$ onto $H_{n,0}(D)$, the closed subspace of $L^2_{n,0}(D)$ consisting of holomorphic (i.e., $\bar{\partial}$-closed) $(n,0)$ forms. We shall say that a smoothly bounded domain D satisfies *condition R* if the Bergman projection associated to D maps $C^\infty_{n,0}(\bar{D})$ into $C^\infty_{n,0}(\bar{D})$. The main result of this paper can now be stated.

THEOREM 1. *Suppose $f: D_1 \to D_2$ is a proper holomorphic mapping between relatively compact, smoothly bounded pseudoconvex domains D_1 and D_2 in n-dimensional Stein manifolds M_1 and M_2, respectively. If D_1 and D_2 satisfy condition R, then f extends smoothly to $\bar{D}_1$.*

REMARKS. A) A domain D is known to satisfy condition R, for example, whenever its associated $\bar{\partial}$-Neumann problem on $(n,0)$ forms is globally regular. For a detailed discussion of the regularity properties of the $\bar{\partial}$-Neumann problem and their relation to the Bergman projection, see J. J. Kohn's papers [7, 8].

B) There is an apparently stronger version of Theorem 1 that can be proved.

THEOREM 2. *Suppose $f: D_1 \to D_2$ is a proper holomorphic mapping between smoothly bounded pseudoconvex domains D_1 and D_2 that are relatively compact inside Stein manifolds of dimension n. If D_1 satisfies condition R, then f extends smoothly to $\bar{D}_1$.*

We shall not prove Theorem 2 here. Our proof of Theorem 1 reveals the basic changes that must be made in the arguments of [4] and [6] to adapt them to

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