APPROXIMATION THEOREMS FOR STRONGLY MIXING RANDOM VARIABLES

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1. Introduction. As a technique for proving limit theorems for dependent random variables, the direct approximation of dependent r.v.'s by independent ones has been gaining popularity since it was introduced in articles by Berkes and Philipp ([2], [3]). The idea is to carry out such an approximation in a manner that permits limit theorems for the independent r.v.'s to carry over directly to the dependent ones. Our purpose is to derive some sharp "approximation theorems" for random sequences satisfying the "strong mixing" condition. Before we discuss some of the results in the literature that pertain to this problem, it will be convenient to define some terminology.

First, a "uniform-[0, 1]" random variable is simply a r.v. which is uniformly distributed on the interval [0, 1].

A "Borel space" is a measurable space $(\mathcal{S}, \mathfrak{D})$ which is (bimeasurably) isomorphic to a Borel subset of the real number line **R**. (No metric is needed in this definition.) When we refer to an "$\mathcal{S}$-valued" r.v. X on a probability space $(\Omega, \mathfrak{F}, P)$, the σ-algebra $\mathfrak{D}$ that accompanies $\mathcal{S}$ will usually be suppressed, but it is implicitly understood that $\forall D \in \mathfrak{D}$ the set of sample points $\{\omega : X(\omega) \in D\}$ is an element of $\mathfrak{F}$. The σ-field of such events $\{X \in D\}$, $D \in \mathfrak{D}$, is denoted by $\mathcal{B}(X)$. The Euclidian spaces $\mathbf{R}^J$, $1 \leqslant J \leqslant \infty$, are always accompanied by the usual (J-dimensional) Borel σ-algebra, and are well known to be Borel spaces.

Let $(\Omega, \mathfrak{F}, P)$ be a probability space. For any two σ-fields $\mathcal{A}$ and $\mathcal{B}$ $(\subset \mathfrak{F})$ define the following measures of dependence

$$\alpha(\mathcal{A}, \mathcal{B}) = \sup |P(A \cap B) - P(A)P(B)| \quad A \in \mathcal{A},\ B \in \mathcal{B}$$

$$\rho(\mathcal{A}, \mathcal{B}) = \sup |\mathrm{Corr}(f, g)| \quad f \in \mathcal{L}^2(\mathcal{A}),\ g \in \mathcal{L}^2(\mathcal{B})$$

$$\beta(\mathcal{A}, \mathcal{B}) = \sup (1/2) \sum_{i=1}^{I} \sum_{j=1}^{J} |P(A_i \cap B_j) - P(A_i)P(B_j)|$$

where this latter sup is taken over all pairs of partitions $\{A_1, \ldots, A_I\}$ and $\{B_1, \ldots, B_J\}$ of Ω such that each $A_i \in \mathcal{A}$ and each $B_j \in \mathcal{B}$. In the definition of $\rho(\mathcal{A}, \mathcal{B})$ it is understood that $\mathrm{Corr}(f, g) \equiv 0$ if f or g is constant a.s. Obviously $\alpha(\mathcal{A}, \mathcal{B}) \leqslant \rho(\mathcal{A}, \mathcal{B}) \leqslant 1$ and $\alpha(\mathcal{A}, \mathcal{B}) \leqslant \beta(\mathcal{A}, \mathcal{B}) \leqslant 1$, but there is no general comparison either way between $\rho(\mathcal{A}, \mathcal{B})$ and $\beta(\mathcal{A}, \mathcal{B})$.

The following approximation theorem comes from Berbee's [1] book.

THEOREM A ([1, Corollary 4.2.5]). *Suppose X and Y are* r.v.*'s taking their values in Borel spaces $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively, and suppose U is a uniform-*[0, 1]

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