DUALITY AND MULTIPLIERS FOR MIXED NORM SPACES

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Introduction. If $p>0$, $q>0$, $\alpha>-1$, a function f, holomorphic in the unit disk U, is said to belong to the space $A^{p,q,\alpha}$ if

$$\|f\|_{p,q,\alpha}^p=\int_0^1 (1-r)^\alpha M_q(r,f)^p\,dr<\infty, \quad \text{where}$$

$$M_q(r,f)^q=\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^q\,d\theta,$$

with the usual understanding if p or $q=\infty$.

In the first section we deal with the dual space of $A^{p,q,\alpha}$. We list some known results and calculate the dual in two cases that, up to now, had been left unsettled.

In the second section we use the duality results of the first section along with a method of J. Anderson and A. Shields, [3], to calculate the coefficient multipliers of some of the spaces $A^{p,q,\alpha}$. It seems that the multiplier theory for $A^{p,q,\alpha}$ is similar to that for H^q. We shall see that if $2\leqslant q\leqslant\infty$ then the multipliers for $A^{p,q,\alpha}$ are the same as for $A^{p,2,\alpha}$. We can find the multipliers for $A^{p,1,\alpha}$ but not for $A^{p,q,\alpha}$, $1<q<2$.

In [2], J. Anderson has calculated the multipliers for Lipschitz spaces Λ_α^p, $2\leqslant p\leqslant\infty$, and asks about similar results for $1\leqslant p<2$. We are able to calculate multipliers for Λ_α^1.

In the third section we introduce some special mixed norm spaces $D^{p,q}$. We show that when $q=2$ these are exactly the spaces D^p introduced by F. Holland and B. Twomey [11]. They showed, using the Hardy–Stein identity, that for $p\leqslant 2$, $H^p\subset D^p$, and for $p\geqslant 2$, $D^p\subset H^p$. Using only a classical result of Hardy and Littlewood we generalize this to show that $H^p\subset D^{p,q}$ for $p<q$ and $D^{p,q}\subset H^p$ for $q<p$. We also give some results on fractional integrals and derivatives for functions in the spaces $D^{p,q}$, as well as a result on multiplication by bounded functions.

1. Duality. If X and Y are spaces of functions holomorphic in U, the statement "$X^*=Y$" means that for every continuous linear form, φ, on X there is a unique $g(z)=\sum_{k=0}^\infty g_k z^k\in Y$ such that if $f(z)=\sum_{k=0}^\infty f_k z^k\in X$ then

$$\varphi(f)=\lim_{r\to 1}\sum_{k=0}^\infty f_k\bar g_k r^k,$$

and conversely if f,g are given as above then $\lim_{r\to 1}\sum_{k=0}^\infty f_k\bar g_k r^k$ exists and defines a bounded linear form on X.

If $g(z)=\sum_{k=0}^\infty g_k z^k$ is holomorphic in U, and α is any real number then we define $(D^\alpha g)(z)=\sum_{k=0}^\infty (k+1)^\alpha g_k z^k$.

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