INTERPOLATION BY FUNCTIONS IN BERGMAN SPACES

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I. Introduction. We begin by describing a special case of our main result. Let U be the upper half plane and let A be the Bergman space consisting of functions f which are holomorphic in U and for which

$$(1.1) \qquad \|f\| = \iint_U |f(x+iy)|\,dx\,dy$$

is finite. Let $S=\{\zeta_n\}$ be a sequence of points in U; $\zeta_n = x_n + iy_n$. We are interested in the relation between the geometry of S and the values which functions in A can take on S. Using the area mean value theorem for a disk centered at $\zeta = x+iy$ in U and having radius y we find that for any f in A

$$(1.2) \qquad |f(\zeta)| \leqslant c\|f\|y^{-2}.$$

Using a similar estimate for derivatives we find that

$$(1.3) \qquad |f'(\zeta)| \leqslant c\|f\|y^{-3}.$$

On the basis of (1.2) we see that the sequence $Tf = \{y_n^2 f(\zeta_n)\}$ is in $l^\infty(S)$. On the basis of the form of (1.1) and the analogy with the known results for the Hardy spaces we then ask for the relation between $l^1(S)$ and $\{Tf; f\in A\}$. (1.3) suggests one constraint. In order for there to be functions f_n in A which satisfy

$$(1.4) \qquad f_n(\zeta_m) = \delta_{n,m}$$

and which have $\|f_n\|$ uniformly bounded it is necessary that

$$(1.5) \qquad \inf_{\substack{n,m\\ n\neq m}} d(\zeta_n, \zeta_m) = K > 0.$$

Here $d(\cdot,\cdot)$ denotes the invariant distance (i.e., the hyperbolic distance) on U.

A particular case of our main result is that this condition is very close to being sufficient.

THEOREM. *There is a K_0 so that if S satisfies (1.5) with $K > K_0$ then $\{Tf; f\in A\} = l^1(S)$. In fact for all f in A*

$$\text{(a)} \qquad \sum_n y_n^2 |f(\zeta_n)| \leqslant c\|f\|$$

and for any $\{\lambda_n\} \in l^1(S)$, there is an f in A with $\|f\| \leqslant c\|\{\lambda_i\}\|$ and

$$\text{(b)} \qquad y_n^2 f(\zeta_n) = \lambda_n \quad n = 1, 2\ldots$$

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