TORSION INVARIANTS AND ACTIONS OF FINITE GROUPS

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Dedicated to the memory of my friend and colleague David L. Williams.

Let G be a finite group and X be a G-CW complex in the sense of T. Matumoto [5] or S. Illman [4]. If X is a finite G-CW complex (i.e., X has only finitely many G-cells), then Illman [4; Section 2] gives a geometric definition for an equivariant Whitehead group $\mathrm{Wh}_G(X)$. Furthermore, he shows [4; Theorem 1.4] that if G is abelian and each component of $X^H=\{x\in X \mid hx=x \text{ for all } h\in H\}$ is simply connected for all subgroups H of G, then $\mathrm{Wh}_G(X)$ is isomorphic to a direct sum of ordinary (i.e., algebraically defined) Whitehead groups. A similar result has been obtained by M. Rothenberg [6; Theorem 1.8].

In a somewhat parallel vein, J. Baglivo [1] considered the following problem. Let X be a G-CW complex which is G-dominated by a finite G-CW complex Y. Does X have the G-homotopy type of a finite G-CW complex? In the approach taken in [1], Baglivo adopts the running hypotheses that $X^G\neq\emptyset$ and that X^H is connected for all subgroups $H\subset G$. Under these conditions, she shows that there exist groups, denoted by $\widetilde{N(H)/H}$ in [1], and elements $w_H(X)\in\tilde{K}_0Z(\widetilde{N(H)/H})$ such that X has the homotopy type of a finite G-CW complex if and only if all the $w_H(X)=0$.

Let X_α^H be a subcomplex of X^H. (In this paper X_α^H will actually be a connectedness component of X^H or a union of such.) Let $G_\alpha=\{g\in G \mid g(X_\alpha^H)=X_\alpha^H\}$ and $N(H)=\{g\in G \mid gHg^{-1}=H\}$ be the normalizer of H. In this paper, we introduce a group $\Gamma(X_\alpha^H,G)$ which fits into a short exact sequence

$$1 \longrightarrow \pi_1(X_\alpha^H) \longrightarrow \Gamma(X_\alpha^H,G) \longrightarrow G_\alpha\cap N(H)/H \longrightarrow 1$$

and use these groups to generalize the results of [1], [4], and [6]. In particular, we establish the following theorems:

THEOREM A. *Let G be a finite group and X be a finite G-CW complex. Let $\{H_s \mid s\in S\}$ be a set of representatives for the subgroups of G that are contained in an isotropy subgroup of the action of G on X. Let $\{X_\alpha^{H_s} \mid \alpha\in A_s\}$ be a set of representatives for the connectedness components of X^{H_s}. Then there exists an isomorphism*

$$\Phi:\mathrm{Wh}_G(X) \longrightarrow \sum_{s\in S}\ \sum_{\alpha\in A_s} \mathrm{Wh}\,\Gamma(X_\alpha^{H_s},G)$$

THEOREM B. *Let G be a finite group and X be a G-CW complex. Let $\{H_s \mid s\in S\}$ be a set of representatives for the set of isotropy subgroups of the action of G on X. Let $\{X_\alpha^{H_s} \mid \alpha\in A_s\}$ be a set of representatives for the connectedness components of X^{H_s}.*

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