A TORUS THEOREM FOR REGULAR BRANCHED COVERS OF S^3

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A closed, orientable, smooth 3-manifold M is said to be a *regular branched cover of* S^3 if there is a finite group G of orientation-preserving diffeomorphisms of M such that the orbit space M/G, regarded as a smooth manifold in the natural way, is diffeomorphic to S^3. The class of 3-manifolds which are regular branched covers of S^3 includes, for instance, the closed, orientable 3-manifolds of Heegaard genus ≤ 2. (This follows from the Birman-Hilden theorem [2]. In this case the group G may be taken to have order 2.)

The proof of the Smith conjecture [1] makes certain hard questions about general 3-manifolds accessible for the class of regular branched covers of S^3. To take a striking example, if the group G in the above definition is cyclic, then it follows from the generalized Smith conjecture (see the Introduction to [1]) that M cannot be simply connected unless it is diffeomorphic to S^3. The main result of this paper is that a strong analogue of the "torus theorem" is true for *all* regular branched covers of S^3.

Let us review the torus theorem in a language convenient for our purposes. For the moment let M be a *prime* 3-manifold. (For the definition of this and other standard terms in 3-dimensional topology, we refer the reader to [6].) By an *essential singular torus* in M we mean a map $f: T^2 \to M$ that induces a monomorphism of fundamental groups. We shall say that *the torus conjecture holds in* M if for every essential singular torus f, $f: T^2 \to M$ is homotopic to a map $g: T^2 \to M$ such that $g(T^2) \subset \Sigma$, where $\Sigma \subset M$ is a compact Seifert fibered space whose boundary components are all incompressible in M. The strongest standard form of the torus theorem, which was proved by Johannson [7, p. 9]; and by Jaco and Shalen [8, p. 55] and which refines results proved by Waldhausen [17] for the bounded case and Feustel [4] in the closed case, asserts that the torus conjecture holds in every *Haken manifold,* i.e. in every compact, orientable, irreducible 3-manifold which contains an incompressible surface. In particular it holds in every bounded, orientable, irreducible 3-manifold.

Now let M be any closed orientable 3-manifold. We shall say that *the torus conjecture holds in* M if it holds in every prime factor of M. We may now state our

MAIN THEOREM. *The torus conjecture holds in every regular branched cover of* S^3.

In Section 1 we interpret results recently proved by Scott, Meeks, Yau and Simon [12], [13], [11], [10] in forms that are useful for our purposes. Scott's results turn out to imply that the torus conjecture holds in manifolds covered by Haken manifolds; we state this in a stronger form, as Theorem 1.2. The Meeks-Yau

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