AN EIGHT-TERM EXACT SEQUENCE ASSOCIATED WITH A GROUP EXTENSION

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Let $1 \to N \to G \to Q \to 1$ be an extension of groups and let A be a G-module. We shall denote by A^N the submodule of A consisting of all N-invariant elements. Hochschild and Serre [4], using spectral sequences, proved that the sequence

$$0 \to H^n(Q,A^N) \to H^n(G,A) \to H^n(N,A)^Q \to H^{n+1}(Q,A^N) \to H^{n+1}(G,A)$$

is exact provided $H^i(N,A) = 0$ for $0 < i < n$. In case $n = 1$, the sequence was extended (to the right) to three more terms by Huebschmann [6] which includes the exact sequence

$$H^2(Q,A^N) \to H^2(G,A) \to \mathrm{XPext}(G,N;A) \to H^3(Q,A^N) \to H^3(G,A)$$

where $\mathrm{XPext}(G,N;A)$ denotes the abelian group of equivalence classes of "crossed pairs." In this paper we show that this can be done even when $n > 1$. More explicitly we prove that if $H^i(N,A) = 0$ for $0 < i < n$, then the following sequence is exact

$$0 \to H^n(Q,A^N) \to H^n(G,A) \to \mathrm{Sext}_G^{n-1}(N,A) \to H^{n+1}(Q,A^N) \to H^{n+1}(G,A)$$
$$\to \mathrm{Sext}_G^n(N,A) \to H^{n+2}(Q,A^N) \to H^{n+2}(G,A)$$

where $\mathrm{Sext}_G^n(N,A)$ denotes the abelian group of equivalence classes of pseudo n-fold extensions of A by N (see Section 2). Note that no spectral sequences are used in the proofs contained in this paper.

In Section 1 we recall the definition of pseudo modules and define pseudo extensions. In Section 2 we derive a long exact sequence of "Sext" in the second variable (which is natural in the first variable). In Section 3 we derive the sequence of Huebschmann to show that it is in fact natural in the variable A. In Section 4 we deduce the main result.

1. PSEUDO MODULES AND PSEUDO EXTENSIONS

Let E be a group with normal subgroup X. Let $\mathrm{Aut}_X E$ denote the group of automorphisms of E which map X onto itself. The subgroup of all automorphisms that "conjugate" by elements of X (i.e. $\phi: E \to E$ such that $\phi(e) = xex^{-1}$ for some x) will be denoted by $c(X)$. If G is a group then a *pseudo G-action* on E is a group homomorphism $\theta: G \to \mathrm{Aut}_X E/c(X)$ for some normal subgroup X of E. For convenience we write $g * e = \hat{\theta}(g)(e)$, $g \in G$, $e \in E$ where $\hat{\theta}(g)$ is a previously chosen element of the coset $\theta(g)$. If $\hat{\theta}_1(g)$ is another element of $\theta(g)$ then $g * e = x\hat{\theta}_1(g)(e)x^{-1}$

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