HOMOGENEOUS, SEPARATING PLANE CONTINUA ARE DECOMPOSABLE

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Dedicated to my mother and father

The problem of identifying the homogeneous plane continua stems from an old question of Knaster and Kuratowski [14]. R. H. Bing [1] and Bing and Jones [3] have made notable inroads on the problem by showing that, in addition to the simple closed curve, the pseudo-arc and the circle of pseudo-arcs are homogeneous plane continua. The list of (nondegenerate) homogeneous plane continua includes only these three.

In 1955, Jones [12] classified the homogeneous plane continua into three types: (1) the ones that do not separate the plane; (2) the decomposable ones that separate the plane; and (3) the indecomposable ones that separate the plane. Continua of types (1) and (3) must be hereditarily indecomposable [9] and [11], while continua of type (2) that are not simple closed curves must admit a continuous decomposition into elements of type (1) such that the resulting quotient space is a simple closed curve [12].

The pseudo-arc is of type (1), while the simple closed curve and the circle of pseudo-arcs are of type (2). No example of a homogeneous plane continuum of type (3) is known. The pseudo-circle of Bing [2], a logical candidate, is known not to be homogeneous [6] or [15].

In this paper, we prove that there do not exist homogeneous continua of type (3).

C. E. Burgess [4, p. 77, Questions 2 and 6] has asked if there exists a homogeneous plane continuum having infinitely many complementary domains, and if there exists, for each positive integer n, a homogeneous plane continuum that separates the plane into n connected domains. It follows from the results of this paper that the answer to both questions is no.

Howard Cook [5] has described, for n a positive integer or ∞, a plane continuum that separates the plane into n complementary domains and has only pseudo-arcs as proper nondegenerate subcontinua. By the results of this paper, none of Cook's continua are homogeneous.

This paper also provides another proof that a pseudo-circle (i.e., a hereditarily indecomposable, circle-like plane continuum different from the pseudo-arc) is not homogeneous.

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