$\mathbf{Z}_2$ SURGERY THEORY

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0. NOTATION AND RESULTS

We want to give a $\mathbf{Z}_2$ surgery theory in a particularly interesting special case, namely allowing fixed point sets up to the middle dimension. This is an extension of Ted Petrie's surgery theory [20] for involutions, whose gap hypothesis would assume for a $\mathbf{Z}_2$ manifold X and each component X^s of $X^{\mathbf{Z}_2}$ that $\dim X^s < 1/2 \dim X$. We shall point out later on in which sense our obstructions differ essentially from the obstructions in Petrie's set-up and the known surgery theory.

The understanding of surgery obstruction theory is a major step in classifying G-manifolds up to diffeomorphism. Some authors [12], [14], [15] have attacked this classification problem using surgery methods in the set-up of the classical work of Kervaire and Milnor [13]. P. Löffler [15] has also obtained results in transformation groups applying obstruction theory to (much less general) surgery problems of the type studied here. Other authors have approached the classification of semilinear actions on homotopy spheres via the study of knot invariants [22], [23] and [24]. In [24], the reader can find a more complete list of references for this problem. We shall study classification problems in a much more general set-up in a later paper by constructing a long exact sequence [8]. Surgery obstruction theory as treated here is a key tool in computing the obstruction group. In this paper we use T. Petrie's G-surgery theory as developed in [18], [19].

One of the crucial problems is doing surgery while leaving a submanifold (here the fixed point set) unchanged. To show that we can do this, we apply the Atiyah-Singer signature theorem in section 2, and in section 4 we use direct computations.

The algebra we use here reflects our geometric situation. We introduce and compute new Wall groups for the group $\mathbf{Z}_2$, which, in a strong sense, are in-between the classical Wall groups [25] and the Witt groups [1]. I should point out that a less general treatment of section 2 appeared in [6].

Notation. All manifolds will be smooth oriented compact $\mathbf{Z}_2$-manifolds, thus the fact is included that the components of the fixed point set are oriented (for an orientable manifold with $\mathbf{Z}_2$ action, it does not follow that the fixed point set is orientable [3]). All maps will be equivariant. Let X and Y be $\mathbf{Z}_2$ manifolds and $f: X \to Y$. Then f is a pseudoequivalence if f is a homotopy equivalence and equivariant.

Definition 0.1. $f: X \to Y$ is an h-normal map if f is of degree 1 and we have a given $\mathbf{Z}_2$ bundle η over Y together with a given trivialization $C: \mathscr{T}X \oplus \varepsilon\, f^* \eta \oplus \varepsilon$.

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