ON THE EXTENSION OF HOLOMORPHIC FUNCTIONS WITH GROWTH CONDITIONS ACROSS ANALYTIC SUBVARIETIES

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1. INTRODUCTION AND STATEMENT OF RESULTS

1.1 In this paper we prove two extension theorems for holomorphic functions in the H^p classes, one for the unit ball $B^n \subset \mathbf{C}^n$, and one for the unit polydisk $U^n \subset \mathbf{C}^n$. We also prove an extension theorem for functions in the Nevanlinna class on B^n. The theorems on the ball can be formulated in the context of an arbitrary bounded domain in $\mathbf{C}^n$ with smooth boundary, as indicated in the course of the proofs. The results are as follows:

THEOREM A. *Let V be an analytic subvariety of B^n. Let f be a holomorphic function on $B^n - V$ such that for some $p > 0$, $|f|^p$ has a harmonic majorant u defined on $B^n - V$. Then f extends to a holomorphic function $\hat{f}$ on B^n which belongs to the class $H^p(B^n)$.*

THEOREM B. *Let V be an analytic subvariety of U^n. Let f be a holomorphic function on $U^n - V$ such that for some $p > 0$, $|f|^p$ has an n-harmonic majorant u defined on $U^n - V$. Then f extends to a holomorphic function $\hat{f}$ on U^n which belongs to the class $H^p(U^n)$.*

THEOREM C. *Let V be an analytic subvariety of B^n. Let f be a holomorphic function on $B^n - V$ such that* $\log^+ |f|$ *has a pluriharmonic majorant u on $B^n - V$. Then f extends to a meromorphic function $\hat{f}$ on B^n which belongs to the Nevanlinna class $N(B^n)$.*

1.2 In the case of the first two theorems, the methods involve extending the majorant u to a superharmonic (respectively n-superharmonic) function $\hat{u}$ on B^n (respectively U^n), and applying the Riesz decomposition theorem for superharmonic functions to obtain a growth estimate for $\hat{u}$. This in turn implies that f has a meromorphic extension to B^n (respectively U^n). The argument is completed by showing that a meromorphic function which is not holomorphic cannot have a harmonic majorant. The fact that the extended function $\hat{f}$ belongs to the appropriate H^p class is a consequence of a property of the integral means of a superharmonic (respectively n-superharmonic) function.

Theorem C uses one-variable methods and the Weierstrass preparation theorem.

1.3 Theorems A, B, and C generalize results of Parreau in one variable [7, Theorem 20] which in fact are formulated in the context of an open Riemann surface and (in place of the zero set of a holomorphic function) a compact subset of logarithmic capacity 0. As far as we know, however, our results in the one-variable

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