JACOBIAN IDEALS AND A THEOREM OF BRIANÇON-SKODA

Joseph Lipman and Avinash Sathaye

1. PRINCIPAL RESULTS

We prove the following generalization of a theorem of Briançon-Skoda (for background cf. [5], [13]):

THEOREM 1. *Let R be a commutative noetherian normal integral domain, and let I_0 be an ideal in R such that the associated graded ring* $\bigoplus_{n\geq 0}(I_0^n/I_0^{n+1})$ *is regular* (i.e., *all its localizations at prime ideals are regular local rings; for example I_0 could be any ideal such that both R/I_0 and the ring of fractions R_{1+I_0} are regular*). *Let I be an ideal of the form $I = I_0 + (y_1, y_2, \ldots, y_{d+1})R$* $(d \geq 0)$ *and let $\lambda \geq 1$ be any positive integer. Then $\overline{I^{d+\lambda}} \subset I^\lambda$ where "$\overline{}$" denotes "integral closure" of an ideal.*

Remarks. (1) Some other versions of Theorem 1 are given at the end of this section.

(2) For $d = 0$, Theorem 1 says that *all powers of I are integrally closed.* For more on this situation see Section 4.

Theorem 1 is a corollary of:

THEOREM 1′. *Let R^* be a commutative noetherian normal integral domain and let $0 \neq t \in R^*$ be such that R^*/tR^* is regular. Let $y_1, \ldots, y_{d+1} \in R^*$, let $S = R^*[y_1/t, \ldots, y_{d+1}/t]$ and let $\bar{S}$ be the integral closure of S (in its field of fractions). Then $t^d\bar{S} \subset S$.*

Indeed, take t to be an indeterminate over R, and set

$$R^* = R[t, I_0 t^{-1}] = \bigoplus_{n\in\mathbf{Z}} I_0^n t^{-n} \qquad (I_0^n = R \quad \text{if} \quad n \leq 0).$$

Then R^* is normal (because each I_0^n is a valuation ideal) and $R^*/tR^* = \bigoplus_{n\geq 0} I_0^n/I_0^{n+1}$ is regular. Now with I as in Theorem 1, the ring S of Theorem 1′ is the graded ring $S = \bigoplus_{n\in\mathbf{Z}} I^n t^{-n}$ $(I^n = R$ if $n \leq 0)$, and so its integral closure is $\bar{S} = \bigoplus_{n\in\mathbf{Z}} \overline{I^n} t^{-n}$. (In fact this is one way to *define* $\overline{I^n}$.). So from Theorem 1′ we conclude that for all n, $\overline{I^n} t^{-n+d} \subseteq I^{n-d} t^{-n+d}$, and setting $n = d + \lambda$ we get Theorem 1.

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