# ON THE BOUND FOR THE DEFRANCHIS-SEVERI THEOREM

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The classical DeFranchis-Severi Theorem (cf. [2], [3], [4]) states: Given a function field of one variable $L$ over an algebraically closed field $K$, then the intermediary extensions $K \subsetneq L' \subset L$ such that genus $(L') \geq 2$ and $L/L'$ is separable, are finitely many in number. A. Weitsman and I raise independently the question whether the number in the above theorem is bounded by some number depending only on the genus of the field $L$. The purpose of the article is to settle the hyperelliptic case. Indeed we have

THEOREM. *Given an integer $g$, there is a number $m_g$ such that for any hyperelliptic field $L$ of genus $g$ over an algebraically closed field of characteristic $\neq 2$, the number $m_g$ is bigger than the number of intermediate field $L'$ with $L/L'$ separable and the genus of $L' \geq 2$.*

The above theorem establishes the credibility of the following conjecture.

CONJECTURE. *The number of subfields in the original DeFranchis-Severi theorem is bounded by some number which depends only on the genus.*

## 1. THE PROOF

Let the genus of $L'$ be $g'$ and $[L:L'] = n$. Then it follows from the Hurwitz formula $2g - 2 = n(2g' - 2) + \delta$ that there are finitely many choices for $g'$ and $n$. Note that $n \leq g - 1$. Thus as usual we may assume that both $g'$ and $n$ are given. Note that $L'$ must be hyperelliptic (cf. [1]).

The canonical map will send $L$ to a rational field $K(x)$ with $[L:K(x)] = 2$. The field $K(x)$ is thus uniquely determined. Let $K(y)$ be the corresponding field for $L'$. Then we have the following diagram

$$\begin{array}{ccccc} & L & \overset{n}{\supset} & L' & \\ 2 & \cup & & \cup & 2 \\ & K(x) & \overset{n}{\supset} & K(y). & \end{array}$$

Let $y = f(x)/g(x)$ and a defining equation of $L'$ over $k(y)$ be $v^2 = \psi(y) = \prod (y - \beta_i)$ where $\psi(y)$ is of degree $2g' + 2$. Then $v \notin k(x)$ and $v$ will generate $L$ over $k(x)$. The above equation can be rewritten as

Received September 1, 1979. Revised version received April 12, 1980.
This work was partially supported by NSF under 0029-50-1395 at Purdue University.
Michigan Math. J. 28 (1981).