A NONZERO-DIVISOR CHARACTERIZATION OF BUCHSBAUM MODULES

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1. INTRODUCTION AND RESULTS

M. Hochster asked the following question in a discussion at the University of Michigan:

Let $A = R/\mathfrak{a}$ be a local ring where R is regular and $\mathfrak{a}$ is an ideal of R. Suppose that

(i) $A_\mathfrak{p}$ is a Cohen-Macaulay ring for all $\mathfrak{p} \in \operatorname{Spec} A \setminus \mathfrak{m}$ where $\mathfrak{m}$ is the maximal ideal of A.

(ii) there exists a nonzero-divisor x of A such that $A/(x)$ is a Buchsbaum ring.

Is it true that then A is a Buchsbaum ring?

Analyzing this question we get a nonzero-divisor characterization of Buchsbaum modules (see theorem) and the Examples 1 and 2 of this note. First we will give the Theorem of this paper.

THEOREM. *Let A be a local ring with maximal ideal* $\mathfrak{m}$ *(A is noetherian, commutative with unit). Let M be a finitly generated and unitary A-module of dimension* $d \geq 1$. *Suppose that* $\operatorname{depth}(M) \geq 1$ *then the following statements are equivalent:*

(i) *M is a Buchsbaum module.*

(ii) *There exists a nonzero-divisor* $x \in \mathfrak{m}$ *for M such that the following conditions are true:*

(a) $M/(x)M$ *is a Buchsbaum module.*

(b) $x \cdot H^i_\mathfrak{m}(M) = 0$ for all $i = 0, \ldots, d-1$.

(ii′) *For every nonzero-divisor* $x \in \mathfrak{m}$ *for M the conditions* (a), (b) *of* (ii) *are true.*

(iii) *There exists a nonzero-divisor* $x \in \mathfrak{m}$ *for M such that the following conditions are true:*

(c) $M/(x)M$ *is a Buchsbaum module.*

(d) $\mathfrak{m} \cdot H^i_\mathfrak{m}(M/(x^2)M) = 0$ *for all* $i = 0, \ldots, d-2$.

(iii′) *For every nonzero-divisor* $x \in \mathfrak{m}$ *for M the conditions* (c), (d) *of* (iii) *are true.*

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