A VANISHING THEOREM FOR CERTAIN RIGID CLASSES

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Let V be a flat n-dimensional vector bundle and X a linear vector field on V which preserves the canonical flat foliation of V. Off a singular set of V, the flat foliation and X determine a codimension $n-1$ foliation and this new foliation has yielded interesting non-vanishing results for exotic classes [3], [6]. But these foliations cannot detect rigid exotic classes because a rigid class is also an exotic class for the codimension n flat foliation, and classes for this foliation vanish for trivial reasons. It is then reasonable to expect that if one takes an appropriate family of more than one linear vector field preserving a flat foliation one would arrive at a new foliation, readily computible in terms of linear data, with some nonzero rigid classes; the above vanishing of rigid classes for trivial reasons no longer holds.

The purpose of this paper is to show that if we take a family of commuting linear vector fields preserving a flat foliation then all rigid classes for the new lower codimension foliation still vanish. This is a companion theorem to those of Pittie [8, the first half of Theorem 2] and Bott-Haefliger [2], all of which assert that all rigid classes for a large family of homogeneous foliation must vanish.

1. DEFINITIONS AND STATEMENT OF THEOREM

For a discussion of exotic classes, see [1]. We will call a foliation *flat* if there is a locally flat basic connection on the normal bundle to the foliation. This is equivalent to having a basic connection ∇ on the normal bundle and a covering family of local framings $\{s_\lambda\}$ of the normal bundle for which $\nabla s_\lambda = 0$. Let us call such a family a family of locally flat framings. A vector field X *preserves* a foliation F if $[X,Y]$ is tangent to F whenever Y is tangent to F. Let X preserve a flat foliation. We will say X is *linear* if there is a family $\{s_\lambda\}$ of locally flat framings for which $L_X(s_\lambda) = A_\lambda s_\lambda$ where A_λ is a constant matrix. Here L_X is the Lie derivative and $L_X(s)$ is the matrix of sections obtained by lifting s to a matrix of vector fields $\tilde{s}$, applying L_X, and projecting back to the normal bundle. This is well defined since X preserves the foliation. Finally we will say that a family of vector fields $\{X_1,\ldots,X_k\}$ is *transverse* to a codimension n foliation if at each point this family and the tangent space to the foliation span a codimension $n-k$ subspace. A family of commuting vector fields $\{X_1,\ldots,X_k\}$ which preserves a codimension n foliation and which is transverse to this foliation determines a codimension $n-k$ foliation; the tangent space to the original foliation and to $\{X_1,\ldots,X_k\}$ is clearly integrable. Our main theorem is then:

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